What is the ratio of their speeds?

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Model Questions

1. The value of

$$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \text{ is}$$

a) 4   b) 0   c) $\sqrt{2}$ d) $3\sqrt{6}$

Sol:

Expression =

$$= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})} -$$

$$- \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} + \frac{\sqrt{6}}{(\sqrt{3}+\sqrt{2})}$$

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(6-2)} +$$

$$\frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}$$

$$= \sqrt{2}(\sqrt{6}-\sqrt{3}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) + \sqrt{6}(\sqrt{3}-\sqrt{2})$$

$$= \sqrt{12} - \sqrt{6} - \sqrt{18} + \sqrt{6} + \sqrt{18} - \sqrt{12}$$

$$= 0 \qquad \textbf{Ans:} \text{ b}$$

2. If cos $2x$ = cos 60° cos 30° + sin 60° sin 30°, then the value of x is

a) 15°   b) 40°
c) 30°   d) None of these

Sol: Cos $2x$ = cos 60° cos 30° + sin 60° sin 30°

$$\Rightarrow \text{Cos } 2x = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$\Rightarrow \cos 2x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos 2x = \cos 30°$$

$$\Rightarrow 2x = 30° \Rightarrow x = 15° \qquad \textbf{Ans:} \text{ a}$$

3. A shopkeeper allows a discount of 10% to his customers and still grains, 20%. Find the marked price of the article which costs Rs. 450?

a) Rs. 600   b) Rs. 540
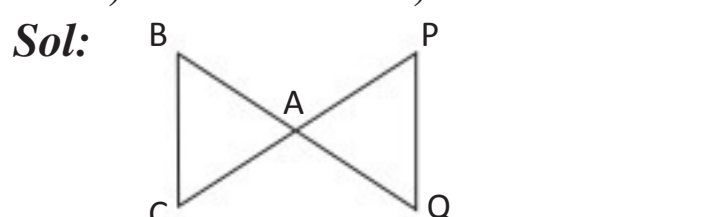c) Rs. 660   d) Rs. 580

Sol: Let the marked price of the article be Rs. x

$$\therefore \quad x \times \frac{90}{100} = \frac{450 \cdot 120}{100}$$

$$\Rightarrow \frac{9x}{10} = 540$$

$$\Rightarrow x = \frac{540 \cdot 10}{9} = Rs. 600$$

Ans: a

4. In the figure $\Delta ACB - \Delta APQ$. If BC = 8 cm, PQ = 4 cm, AP = 2.8 cm, Find CA:

a) 8 cm   b) 6.5 cm
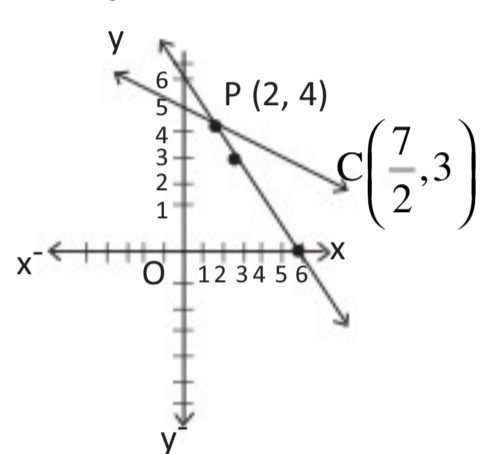c) 5.6 cm   d) None of these

Sol:

$$\frac{BC}{PQ} = \frac{AC}{AP} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{8}{4} = \frac{AC}{2.8} \Rightarrow AC = 5.6 \text{ cm}$$

Ans: c

5. When we draw the graphs of the equations $x + y = 6$ and $2x + 3y = 16$ on the same graph paper, the coordinates of the point where the two lines intersect are

a) (−3, 6)   b) (−2,0)
c) (2, 4)   d) (1, 3)

Sol: If $x + y = 6m$, then $y = 6 - x$

For $x = 0$, $y = 6$
$x = 2$, $y = 4$

If $2x + 3y = 16$ then

$$y = \frac{16 - 2x}{3}$$

For, $x = 2$, $y = 4$
$x = \frac{7}{2}$, $y = 3$

Ans: c

6. The mean of 50 numbers is 30. Later it was discovered that two entries were wrongly entered as 82 and 13 instead of 28 and 31. Find the correct mean

a) 36.12   b) 30.66
c) 29.28   d) 38.21

Sol:

Required average

$$= 30 + \frac{(28+31-82-13)}{50}$$

$$= 30 + \left(-\frac{36}{50}\right) = 30 - 0.72 = 29.28$$

Ans: c

7. Krishna purchased a number of articles at Rs. 10 for each and the same number for Rs. 14 each. He mixed them together and sold them for Rs. 13 each. Then his gain or loss percent is

a) Loss $8\frac{1}{3}\%$ b) Gain $8\frac{2}{3}\%$

c) Loss $8\frac{2}{3}\%$ d) Gain $8\frac{1}{3}\%$

Sol: Let 10 articles of each kind be bought

$\therefore$ Total cost
= Rs. (10×10+14× 10) = Rs. 240
Total selling price
= 13×20 = Rs. 260
$\therefore$ Gain percent

$$= \frac{260-240}{240} \cdot 100$$

$$= \frac{20 \cdot 100}{240} = 8\frac{1}{3}\% \qquad \textbf{Ans:} \text{ d}$$

8. Find a point on the y−axis which is equidistant from the points A (6, 5) and B (−4, 3)

a) (0, 9)   b) (9, 0)
c) (3, 0)   d) (4, 0)

Sol: Let the required point be P (o, y)
$\therefore$ PA = PB

$$\Rightarrow \sqrt{(0-6)^2 + (y-5)^2}$$

$$\Rightarrow \sqrt{(0+4)^2 + (y-3)^2}$$

$$\Rightarrow 36 + (y-5)^2 = 16 + (y-3)^2$$

$$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$$

$$\Rightarrow 4y = 36 \Rightarrow y = 9$$

$\therefore$ Required point is (0, 9).

Ans: a

9. If 90% of A = 30% of B and B = $2x\%$ of A, then the value of x is

a) 450   b) 400
c) 300   d) 150

Sol: $\frac{A \cdot 90}{100} = \frac{B \cdot 30}{100}$

$$\Rightarrow 3A = B$$

$$\Rightarrow 3A = A \times 2x/100$$

$$\Rightarrow 300 = 2x \Rightarrow x = 150$$

Ans: d

10. A man stands at a point A on the bank of a river and looks at the top of a tree which is exactly opposite to him on the other bank. The angle of elevation is 45°. He then walks 200 m at right angles to the bank and away from it to the point B. From B he looks at the top of the tree and the angle of elevation as 30°. The height of the tree is:

a) 10($\sqrt{3}$ + 1)m
b) 100($\sqrt{3}$ −1)m
c) 88 ($\sqrt{3}$ +1)m
d) 100($\sqrt{3}$+1)m

Sol: $\angle ATE = 90°$
$\therefore \angle AET = 45°$
Hence AT = TE, where TE represents the tree
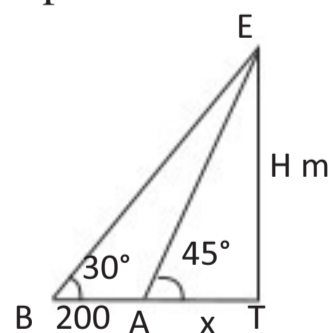
Let AT = TE = x m

From Δ BTE, $\frac{ET}{BT} = \tan 30°$

$$\Rightarrow \frac{x}{x+200} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \frac{200}{\sqrt{3}-1} = \frac{200 \cdot \sqrt{3}+1}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= 100 (\sqrt{3} + 1) \qquad \textbf{Ans:} \text{ d}$$

11. A sum of money placed at compound interest doubles itself in 4 years. In how many years will it amount to four times itself?

a) 12 years   b) 13 years
c) 8 years   d) 16 years

Sol:

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$\Rightarrow 2 = 1\left(1+\frac{R}{100}\right)^4$$

$\therefore$ both side square

$$\Rightarrow 2^2 = \left(1+\frac{R}{100}\right)^8$$

$\therefore$ After 8 years the principle will be 4 times itself. **Ans:** c

12. Ravi purchased an article marked at Rs. 21000 at 5% discount. The rate of sales tax on the article is 10%. Money paid by Ravi to purchase the article is

a) Rs. 19,845   b) Rs. 19,950
c) Rs. 23,100   d) Rs. 21,945

Sol: Price of article after discount

$$= Rs. \left(\frac{21000 \cdot 95}{100}\right) = Rs. 19950$$

Price of article after sales tax

$$= Rs. \left(\frac{19950 \cdot 110}{100}\right) = Rs. 21945$$

Ans: d

13. A kinetic Honda covers a distance of 140 km in 2 hours and 20 minutes while a car covers the same distance in 1 hour and 40 minutes. What is the ratio of their speeds?

a) 6 : 7   b) 3 : 7
c) 4 : 7   d) 5 : 7

Sol: Speed = $\frac{Distance}{Time}$

$\therefore$ Required ratio

$$= \frac{140km}{140 \text{ minutes}} : \frac{140km}{100 \text{ minutes}}$$

$$= 100 : 140 = 5 : 7 \qquad \textbf{Ans:} \text{ d}$$

14. In a mixture of 35 liters, the ratio of milk and water is 4 : 1. How many liters of water must be added to make the ratio 2 : 3?

a) 70   b) 28
c) 40   d) 35

Sol: In 35 liters of mixture,

Milk = $\frac{4}{5} \times 35 = 28$ liters

Water = 7 liters
It x liters of water be added, then

$$\frac{28}{x+7} = \frac{2}{3}$$

$$\Rightarrow 2x + 14 = 28 \times 3 = 84$$

$$\Rightarrow 2x = 84 - 14 = 70$$

$$\Rightarrow x = \frac{70}{2} = 35 \text{ liters} \qquad \textbf{Ans:} \text{ d}$$

15. A and B together can complete a work in 12 days. A alone can complete it in 20 days. If B now does the work only for half a day daily, then the number of days required to complete the work, by A and B together, is

a) 15   b) 14
c) 16   d) 18

Sol: B's 1 day's work

$$= \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60} = \frac{1}{30}$$

Hence, B alone will complete the work in 30 days
On working for half a day, daily, time taken by B = 60 days

$$\Rightarrow \frac{1}{20} + \frac{1}{60}$$

$$\Rightarrow \frac{3+1}{60} \Rightarrow \frac{4}{60} \Rightarrow \frac{1}{15} = 15 \text{ days}$$

Ans: a

16. A pump can fill a cistern in 40 minutes and another pump can empty the filled up cistern in 1 hour. By mistake, with out closing the second pump, the first pump is opened. The time in which the empty cistern will be filled up is

a) 4 hours   b) 1 hour
c) 2 hours   d) 3 hours

Sol: Part of the cistern filled in 1 hour when both pumps are opened together

$$= \frac{1}{40} - 1 = \frac{60}{40} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$$

$\therefore$ Required time = 2 hours

Ans: c

17. The diameter of a roller is 84 cm and its length is 120 cm. It makes 500 complete revolutions to move once over to level a playground. Find the area of the playground in sq. cm. $\left(\pi = \frac{22}{7}\right)$

a) 1594 sq. m   b) 1580 sq. m
c) 1584 sq. m.   d) 1590 sq. m.

Sol: Area of the playground leveled in one revolution
= $2\pi r \times$ length

$$= 2 \times \frac{22}{7} \times 42 \times 120 = 31680 \text{ sq. m.}$$

$\therefore$ Area of the playground
= (500 × 31680) sq.cm
= 15840000 sq.cm
= 1584 sq. metre **Ans:** c

18. The curved surface area of a right circular cylinder of radius 2 cm is 264cm². The height of the cylinder, in cm, is

a) 14   b) 10.5
c) 21   d) 42

Sol: Curved surface area of cylinder
= $2\pi rh$
$\Rightarrow 2\pi rh = 264$

$$\Rightarrow 2 \cdot \frac{22}{7} \cdot 2 \cdot h = 264$$

$$\Rightarrow h = \frac{264 \cdot 7}{2 \cdot 22 \cdot 2} = 21 \text{ cm}$$

Ans: c

19. A rides from his house at 12 noon towards B's house at a speed of 8 km/hour. After 1 hour, B rides from his house towards A's at a speed of 7 km/hour. If the distance between the two houses is 68 km, the time when they meet is

a) 6 PM   b) 3 PM
c) 4 PM   d) 5 PM

Sol: Let both meet after t hours.
$\therefore$ Distance covered by A in t hours + distance covered by B in (t−1) hours = 68 km
$\Rightarrow 8t + 7(t−1) = 68$
$\Rightarrow 8t + 7t − 7 = 68$
$\Rightarrow 15t = 68 + 7 = 75$

$$\Rightarrow t = \frac{75}{15}$$

= 5 hours i.e. at 5 p.m. **Ans:** d