The height of the centre of the balloon is..

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JEE Main Mathematics Special

MODEL QUESTIONS

1. A round balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its center is β. The height of the centre of the balloon is

1) $r\sin\alpha\cos ec\left(\frac{\beta}{2}\right)$
2) $r\sin\beta\cos ec\left(\frac{\alpha}{2}\right)$
3) $r\sin\alpha\sec\left(\frac{\beta}{2}\right)$
4) $r\sin\beta\sec\left(\frac{\alpha}{2}\right)$

2. The distance of the point (2,1,-2) from the line $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-3}$ measured parallel to the plane $x+2y+z=4$ is

1) $\sqrt{10}$ 2) $\sqrt{20}$
3) $\sqrt{5}$ 4) $\sqrt{30}$

3. $\int\frac{5x^4+4x^5}{(x^5+x+1)^2}dx$

1) $x^5+x+1+c$
2) $\frac{x^5}{x^5-x+1}+c$
3) $x^{-4}+x^{-5}+c$
4) $\frac{x^5}{x^5+x+1}+c$

4. The population of a country increased at a rate proportional to the number of in habitants. If the population which doubles in 30 years, then the population will triple in approximately.

1) 30 years 2) 45 years
3) 48 years 4) 54 years

5. The value of integral

$$\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$$

is equal to

1) π 2) 2π
3) 4π 4) π/2

6. The area bounded by the curve $y=2x-x^2$ and the straight line $y=-x$ is given by

1) $\frac{9}{2}$ 2) $\frac{43}{6}$
3) $\frac{35}{6}$ 4) $\frac{13}{6}$

7. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is (¾). If the signal received at station B is green, Then the probability that the original signal was green, is

1) $\frac{3}{5}$ 2) $\frac{6}{7}$
3) $\frac{20}{3}$ 4) $\frac{9}{23}$

8. If p : 4 is an even prime number q : 6 is a divisor of 12 and r: the HCF of 4 and 6 is 2, then which one of the following is true?

1) $p\wedge q$ 2) $(p\vee r)\wedge\sim r$
3) $\sim(q\wedge r)\wedge p$ 4) $\sim p\vee(q\wedge r)$

9. Length of latus rectum of ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$, if the normal at an end of latus rectum passes through one extremity of the minor axis, is

1) 2–√3 2) 6–√3
3) 6–2√3 4) 6 – 2√5

10. α, β, γ are the angles made by a line with x, y z axes in positive direction then the range of $\cos\alpha\cos\beta+\cos\beta\cos\gamma+\cos\gamma\cos\alpha$ is

1) $\left[\frac{-1}{2},1\right]$ 2) $\left[\frac{-1}{2},\alpha\right]$
3) (1, α) 4) (1, 2]

11. Given that matrix $A=\begin{bmatrix}x&3&2\\1&y&4\\2&2&z\end{bmatrix}$. If $xyz=2013$ and $8x+4y+3z=2012$ then A(adj A) is equal to

1) $\begin{bmatrix}68&0&0\\0&68&0\\0&0&68\end{bmatrix}$ 2) $\begin{bmatrix}34&0&0\\0&34&0\\0&0&34\end{bmatrix}$
3) $\begin{bmatrix}29&0&0\\0&29&0\\0&0&29\end{bmatrix}$ 4) $\begin{bmatrix}58&0&0\\0&58&0\\0&0&58\end{bmatrix}$

12. A particle acted by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is

1) 30 units 2) 40 units
3) 50 units 4) 20 units

13. It is known that $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$. Then $\sum_{r=1}^{\infty}\frac{1}{r^2}$ is equal to

1) $\frac{\pi^2}{24}$ 2) $\frac{\pi^2}{3}$
3) $\frac{\pi^2}{6}$ 4) None

14. The no. of points of discontinuity of $f(x)=[x]+\left[x+\frac{1}{4}\right]+\left[x+\frac{1}{2}\right]+\left[x+\frac{3}{4}\right]$ in (0,1] where ([.] denotes Greatest integer function)

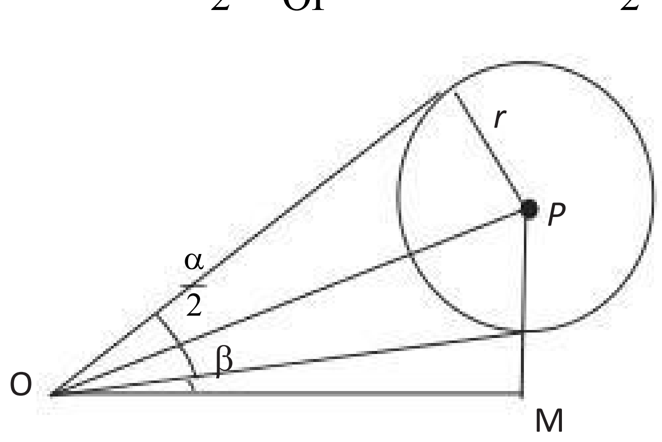
1) 4 2) 2 3) 0 4) 8

15. Let x = {1, 2, 3, 4, 5}. The number of different ordered pairs (y, z) that can be formed such that $y\in x$, $z\in x$ and $y\cap z$ is empty is

1) 5^2 2) 3^5
3) 2^3 4) 5^3

KEY & HINTS

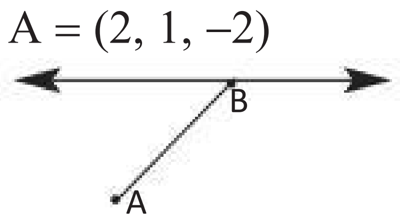
1. 2; $\sin\frac{\alpha}{2}=\frac{r}{OP}\Rightarrow OP=r\cos ec\frac{\alpha}{2}$



$\sin\beta=\frac{PM}{OP}$

$\Rightarrow PM=OP\sin\beta=r\sin\beta\cos ec\frac{\alpha}{2}$

2. 4;
A = (2, 1, –2)



Line: $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-3}$

B is a point on the given line and AB is parallel to given plane

$B=(1+2\lambda,-1+\lambda,3-3\lambda)$

D.r's of

$\overline{AB}=(2\lambda-1,\lambda-2,-3\lambda+5)$

$1(2\lambda-1)+2(\lambda-2)+1(-3\lambda+5)=0$

$\Rightarrow\lambda=0$

B = (1, –1, 3)

3. 4;

$I=\int\frac{\left(\frac{5}{x^6}+\frac{4}{x^5}\right)}{\left(1+\frac{1}{x^4}+\frac{1}{x^5}\right)^2}dx$ and

put $t=1+\frac{1}{x^4}+\frac{1}{x^5}$

4. 3;

$\frac{dp}{dt}=Kp\Rightarrow p=ce^{kt}$

if when t = 0, $p=p_0$ then $p=p_0e^{kt}$ given when t = 30, $p=2p_0$
$\Rightarrow ln_2=30k$

$\therefore$ For $3=e^{kt}\Rightarrow in3=kt$

$\therefore t=\frac{\log 3}{\log 2}.30$

$xt\cong 48$

5. 1;

$\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$

$=\int_{-1}^{0}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$

$+\int_{0}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$

$=\int_{-1}^{0}-\frac{\pi}{2}dx+\int_{0}^{3}\frac{\pi}{2}dx$

$=\left[-\frac{\pi}{2}x\right]_{-1}^{0}+\left[\frac{\pi}{2}x\right]_{0}^{3}=\pi$

6. 1;

Area=

$\int_{0}^{3}(2x-x^2+x)dx=\frac{3x^2}{2}-\frac{x^3}{3}\Big|_{x=0}^{x=3}$

$=\frac{27}{6}=\frac{9}{2}$

7. 3;

Consider the events E1,E2,Ei and G

E_1= A receives the signal correct
E_2= B receives the signal correct
Ei= signal received by B is green
G=Original signal is green

$P(E_1)=\frac{3}{4},P(E_2)=\frac{3}{4}$

$P(\overline{E}_1)=\frac{1}{4},P(\overline{E}_2)=\frac{1}{4}$

$P(G)=\frac{4}{5},P(\overline{G})=\frac{1}{5}$

$P(E)=P(GE_1E_2)+P(G\overline{E}_1\overline{E}_2)+P(\overline{G}E_1\overline{E}_2)+P(\overline{G}\overline{E}_1E_2)$

$P(E)=\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}$

$=\frac{36+4+3+3}{80}=\frac{46}{80}$

Required probability,

$P(G/E)=\frac{P(GE_1E_2)+P(G\overline{E}_1\overline{E}_2)}{P(E)}$

$=\frac{\frac{36}{80}+\frac{4}{80}}{\frac{46}{80}}=\frac{20}{23}$

8. 4;
P is False; q is true; r is true

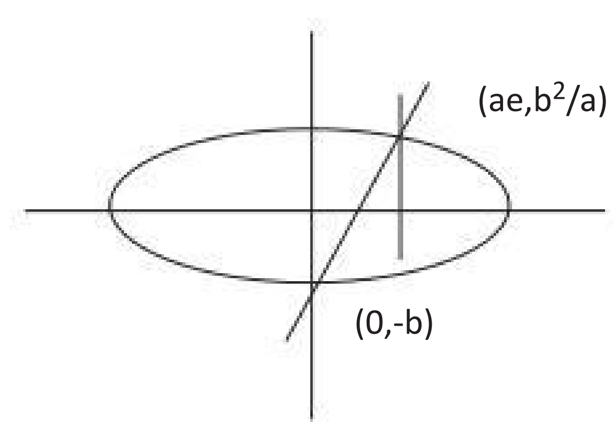
9. 4;
Normal at (ae, b^2/a) pass through (0, –b)

$\Rightarrow e^4+e^2-1=0\Rightarrow e^2=\frac{\sqrt{5}-1}{2}$

also

$b^2=a^2(1-e^2)=4\left(1-\frac{\sqrt{5}-1}{2}\right)$

$=2(3-\sqrt{5})$



Length of L.R.

$=\frac{2b^2}{a}=\frac{4(3-\sqrt{5})}{2}=6-2\sqrt{5}$

10. 1;

$\ell=\cos\alpha,\ m=\cos\beta,\ n=\cos\gamma$

$\ell^2+m^2+n^2=1$ and $(\ell+m+n)^2\geq 0$

$\ell^2+m^2+n^2+2(\ell m+mn+n\ell)\geq 0$

$\Rightarrow\ell m+mn+n\ell\geq\frac{-1}{2}$ and

$\Rightarrow\frac{1}{2}(\ell^2+m^2+n^2-\ell m-mn-\ell n)\geq 0$

$\Rightarrow\ell m+mn+n\ell\geq\frac{-1}{2}\Rightarrow\left[\frac{-1}{2},1\right]$

11. 3;

$A(adjA)=|A|I;\ \ |A|=29$

12. 2;

Here

$\vec{F}=\vec{F}_1+\vec{F}_2=(4\hat{i}+\hat{j}-3\hat{k})+(3\hat{i}+\hat{j}-\hat{k})$

$=7\hat{i}+2\hat{j}-4\hat{k}$ and

$\vec{d}=\vec{d}_2-\vec{d}_1=(5\hat{i}+4\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$

$=4\hat{i}+2\hat{j}-2\hat{k}$

$\therefore$ work done

$=\vec{F}.\vec{d}=(7\hat{i}+2\hat{j}-4\hat{k}).(\hat{i}+2\hat{j}-2\hat{k})$

= (7)(4) + (2)(2)+ (–4)(–2)

= 28 + 4 +8 = 40 units.

13. 3;

Here,

$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......\infty=\frac{\pi^2}{8}$

Let $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.......\infty=x$

Then, $x=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+.......\infty$

$=\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......\infty\right)+$

$+\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+.......\infty\right)$

$=\frac{\pi^2}{8}+\frac{1}{4}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....\infty\right)$

$=\frac{\pi^2}{8}+\frac{1}{4}x$

$\Rightarrow\frac{3x}{4}=\frac{\pi^2}{8}\Rightarrow x=\frac{\pi^2}{6}$

14. 1;

$f(x)=[x]+[x+¼]+\left[x+\frac{2}{4}\right]$

$+[x+¾]=[4x]$

- 14. is disc. At integers i.e.
- 4x = 1
- 4x = 2
- 4x = 3

4x = 4

Number of disc. points is 4

15. 2;

$x_1\in y,\ x_1\in z$
$x_1\notin y,\ x_1\in z$
$x_1\in y,\ x_1\notin z$ $\quad y\cap z=\phi$
$x_1\notin y_1,\ x_1\notin z$

Each $x_1\in X$ having 3 chances

$n(y\cap z)=3^5$