Find the equation of the curve passing through..

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JEE Main Mathematics Special

MODEL QUESTIONS

1. Let p and q be the roots of equation $x^2 - 2x + A = 0$ and let 'r' and 's' be the roots of the equation $x^2 + 18 + b = 0$ if p < q < r < s are in A.P, then value of 'A', 'B' are ___
1) A = 3, B = 77
2) A = 3, B = 7
3) A = – 3, B = 77
4) A = 3, B = – 7

2. The equation of the tangent, to the curve $y = e^{-|x|}$ at the point where the curve cuts the line x = 1, is
1) $x + y = e$ 2) $e(x + y) = 1$
2) $y + ex = 1$ 4) $x - ey = 0$

3. If the sum of squares of roots of equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ is least then α =
1) 90° 2) 70°
3) 20° 4) 60°

4. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c, d are the length of the sides of the quadrilateral, then $a^2 + b^2 + c^2 + d^2$ lies in
1) [1,3] 2) [1,4]
3) [2,3] 4) [2,4]

5. A line through A (–5, –4) meets the line $x + 3y + 2 = 0$, $2x + y + 4 = 0$ and $x - y - 5 = 0$ at B, C and D respectively.
If $\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$,
then the equation of the line is
1) $2x + 3y + 22 = 0$
2) $5x - 4y + 7 = 0$
3) $3x - 2y + 3 = 0$
4) None of these

6. The weighted mean of first n natural numbers whose weights are equal to the number of selections out of n natural numbers of corresponding numbers respectively is
1) $\frac{n.2^{n-1}}{2^n - 1}$ 2) $\frac{3n(n+1)}{2(2n+1)}$
3) $\frac{(n+1)(2n+1)}{6}$
4) $\frac{n(n+1)}{2}$

7. The distance between the line $\bar{r} = 2i - 2j + 3k + \lambda(i - j + 4k)$ and the plane $r.(i + 5j + k) = 5$
1) $\frac{10}{3\sqrt{3}}$ 2) $\frac{10}{9}$
3) $\frac{10}{3}$ 4) $\frac{3}{10}$

8. Find the equation of the curve passing through (1, 2) whose differential equation is $y(x + y^3)\,dx = x(y^3 - x)dy$
1) $xy = 1$ 2) $x^2 - y^2 = 1$
3) $y^3 + 2x = 5x^2$
4) $x^2 - y + 3 = 0$

9. Let a = 111…..1 (55 digits), b = 1 + 10 + 10^2 + ….10^4, c = 1 + 10^5+ 10^{10} + 10^{15} + … + 10^{50}, then
1) a = b + c 2) a = bc
3) b = ac 4) c = ab

10. The area of the region bounded by the curves $y = 9x^2$ and $y = 5x^2 + 4$(in square units) is
1) $\frac{16}{3}$ 2) $\frac{64}{3}$
3) $\frac{32}{3}$ 4) 64

11. Let A be the set of all determinants of order 3 with entries 0 or 1 only, B is the subset of A consisting of all determinants with value 1, and C is the subset consisting of all determinants with value –1. Then if n (B) and n(C) denote the number of elements in B and C, respectively, we have
1) C = ϕ 2) n(B) = n(C)
3) A = B ∪ C 4) n(B) = 2n(C)

12. The sum of two positive integers is 200 then chance that their product is greater than ¾ times their greatest product probability is
1) $\frac{51}{99}$ 2) $\frac{99}{199}$
3) $\frac{1}{2}$ 4) $\frac{1}{3}$

13. If $\int \frac{(2x^2+1)\,dx}{(x^2-4)(x^2-1)} = \log\left[\left(\frac{x+1}{x-1}\right)^a\left(\frac{x-2}{x+2}\right)^b\right] + C$, then the values of a and b are respectively
1) ½, ¾ 2) –1, 3/2
3) 1, 3/2 4) –1/2, 3/2

14) $\int_0^{\pi/2} \frac{\sin^2 9x}{\sin x} dx =$
1) $1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9}$
2) $\frac{1}{2} + \frac{1}{4} + ... + \frac{1}{18}$
3) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + ... + \frac{1}{19}$
4) $1 + \frac{1}{3} + \frac{1}{5} + ... + \frac{1}{17}$

15. If $f(x) = \begin{cases} x^\alpha \log x, x > 0 \\ 0 \quad , x = 0 \end{cases}$ and Rolle's theorem is applicable to f(x) for x [0,1] then is equal to
1) –2 2) –1
3) 0 4) ½

16. The expression $\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7 - \left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$ is a polynomial in x of degree
1) 7 2) 5
3) 4 4) 3

17. A tangent is drawn to the circle $2(x^2 + y^2) - 3x + 4y = 0$ and it touches the circle at point 'A'. The tangent passes through the point P(2, 1). Then PA =
1) 4 2) 2
3) 2√2 4) 8

18. Range of values K of for which the point (k, –1) is exterior to both the parabolas $y^2 = |x|$ is
1) (–1, 0) 2) (–1, 1)
3) (0, 1) 4) (0, –1)

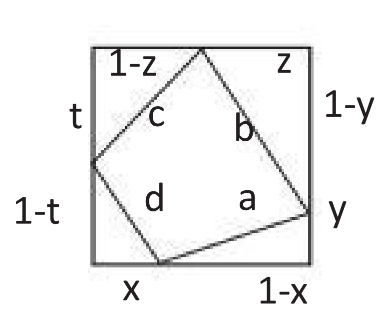
KEY & HINTS

1. 3;
Let roots are
$a - 3d, a - d, a + d, a + 3d$
$p + q = 2,\ pq = A,\ r + s = 18,\ rs = B$
$p + q + r + s = 4a = 20 \Rightarrow a = 5$
$p + q = a - 3d + a - d \Rightarrow d = 2$
Numbers are –1, 3, 7, 11
pq = – 3 = A
B = rs = 77

2. 4;
Point of intersection is $\left(1, \frac{1}{e}\right)$,
and slope = $\frac{1}{e}$

3. 1;
$S = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= \sin^2\alpha - 2\sin\alpha + 5$
$= (\sin\alpha - 1)^2 + 5 \Rightarrow \alpha = 90°$

4. 4;
$a^2 + b^2 + c^2 + d^2$
$= \sum(1-x)^2 + y^2 = \sum x^2 + (1-x)^2$
$x^2 + (1-x)^2$ attains maximum value 1 for x = 0, 1



For $x^2 + (1 - x)^2$ minimum value ½ for x = ½
$\therefore 2 \le a^2 + b^2 + c^2 + d^2 \le 4$

5. 1;
The parametric equation of a line passing through A is
$\frac{x+5}{\cos\theta} = \frac{y+4}{\sin\theta} = r$
let $AB = r_1, AC = r_2, AD = r_3$
$B(-5 + r_1\cos\theta, -4 + r_1\sin\theta)$,
$C(-5 + r_2\cos\theta, -4 + r_2\sin\theta)$,
$D(-5 + r_3\cos\theta, -4 + r_3\sin\theta)$
Sub in given lines.
We get $\frac{15}{r_1} = \cos\theta + 3\sin\theta$

6. 1;
The required mean
$\bar{X} = \frac{1.{}^nC_1 + 2.{}^nC_2 + 3.{}^nC_3 + ... + n.{}^nC_n}{{}^nC_1 + {}^nC_2 + ... + {}^nC_n}$
$= \frac{\sum_{r=0}^{n} r.{}^nC_r}{\sum_{r=1}^{n} {}^nC_r} = \frac{\sum_{r=1}^{n} r.\frac{n}{r}{}^{n-1}C_{r-1}}{\sum_{r=1}^{n} {}^nC_r} = \frac{n\sum_{r=1}^{n} {}^{n-1}C_{r-1}}{\sum_{r=1}^{n} {}^nC_r}$
$= \frac{n(2^{n-1})}{(2^n - 1)}$

7. 1;
Distance $= \frac{\bar{a}.\bar{n} - \bar{d}}{|\bar{n}|}$
$= \frac{(2i + 2j + 3k).(i + 5j + k) - 5}{\sqrt{1^2 + 5^2 + 1^2}} = \frac{10}{3\sqrt{3}}$

8. 3;
$-x^2y^2.\frac{xdy - ydx}{x^2} + x(ydx + xdy) = 0$
$\frac{-y}{x}d\left(\frac{y}{x}\right) + \frac{dxy}{x^2y^2} = 0$
On integration $-\frac{\left(\frac{y}{x}\right)^2}{2} - \frac{1}{xy} = c$
passes through (1, 2) $\Rightarrow c = \frac{-5}{3}$
$y^3 + 2x - 5x^2y = 0$

9. 2;
$a = 1 + 10 + 10^2 + ... + 10^{54}$
$\frac{10^{55} - 1}{10 - 1} = \frac{10^{55} - 1}{10^5 - 1}\cdot\frac{10^5 - 1}{10 - 1} = bc$

10. 1;
$\left.\begin{array}{l} y = 9x^2 \\ y = 5x^2 + 4 \end{array}\right\}$ $4x^2 - 4 = 0$, $\Delta = 64$
Area $= \frac{\Delta^{3/2}}{6a^2} = \frac{8 \times 8 \times 8}{6 \times 4 \times 4} = \frac{16}{3}$

11. 2;
Since, C cannot be the empty set, Hence
$-1 = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \in C$
We also have
$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 2$
So A ≠ B ∪ C. In general, the determinant
$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$
$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{31}a_{22}$
With the a's being 0 or 1 , equals 1 only if $a_{11}a_{22}a_{33} = 1$ and the remaining terms are zero; if $a_{12}a_{23}a_{31} = 1$ and the remaining terms are zero; or if $a_{13}a_{21}a_{32} = 1$ and the remaining terms are zero. Since there are three similar relations for determinants that equal –1 , we must have N(B) = n(C)

12. 3;
$x(200 - x) > \frac{3}{4} \times 100 \times 100 \Rightarrow 50 < x < 150$
$\Rightarrow P(E) = \frac{50}{100} = \frac{1}{2}$

13. 1;
$I = \int \frac{(2x^2 + 1)}{(x^2 - 4)(x^2 - 1)} dx$
$\frac{2x^2 + 1}{(x^2 - 4)(x^2 - 1)} = \frac{3}{(x^2 - 4)} - \frac{1}{x^2 - 1}$
$\therefore I = \int\left[\frac{3}{(x^2 - 4)} - \frac{1}{x^2 - 1}\right]dx$
$= \frac{3}{2 \times 2}\log\left|\frac{x-2}{x+2}\right| - \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$
$= \frac{3}{4}\log\left|\frac{x-2}{x+2}\right| + \log\left|\frac{x+1}{x-1}\right|^{1/2} + c$
$= \log\left|\frac{x-2}{x+2}\right|^{3/4} + \log\left|\frac{x+1}{x-1}\right|^{1/2} + c$
$= \log\left[\left(\frac{x+1}{x-1}\right)^{1/2}\left(\frac{x-2}{x+2}\right)^{3/4}\right] + c$

14. 4; Observe that
$\sin x + \sin 3x + \sin 5x + ... + \sin 17x = \frac{\sin^2 9x}{\sin x}$
$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin^2 9x}{\sin x} dx = -\left(\cos x + \frac{\cos 3x}{3} + \frac{\cos 5x}{5} + ... + \frac{\cos 17x}{17}\right)_0^{\frac{\pi}{2}}$

15. 4;
Since, 'f' is continuous, differentiable
$\underset{x \to 0}{Lt}\ x^\alpha \log x = f(0)$
[f is continuous]
$\underset{x \to 0}{Lt}\ \frac{\log x}{x^{-\alpha}} = 0$
$\underset{x \to 0}{Lt}\ \frac{1/x}{-\alpha.x^{-\alpha-1}} = 0$
$\underset{x \to 0}{Lt}\ \frac{-1}{\alpha}\cdot\frac{1}{x^{-\alpha}} = 0$
$\Rightarrow \underset{x \to 0}{Lt}\ \frac{-1}{\alpha}.x^\alpha = 0$
$\Rightarrow \boxed{\alpha = ½}$

16. 4;
Put $\sqrt{4x + 1} = y$ and expand, then highest power is y^6

17. 2;
$S = x^2 + y^2 - \frac{3}{2}x + 2y$
$PA = \sqrt{S_{11}} = 2$

18. 2;
The two parabolas are
$y^2 = x$ and $y^2 = -x$
The point (k, –1) is an exterior point.
if $(-1)^2 - k > 0$ and
$(-1)^2 + k > 0$
Hence Range of
K is (–1, 1)