The variance of the combined data set is..

M.N. Rao
Senior faculty, Sri Chaitanya Educational institutions

JEE Main Mathematics Special

MODEL QUESTIONS

1. If Z is a non-real complex number, then the minimum value of $\frac{\text{Im}Z^5}{\text{Im}^5 Z}$ is
1) –1 2) –2
3) –4 4) –5

2. $\underset{x\to 1^-}{\text{Lt}} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to:
1) $\frac{1}{\sqrt{2\pi}}$ 2) $\sqrt{\frac{2}{\pi}}$
3) $\sqrt{\frac{\pi}{2}}$ 4) $\sqrt{\pi}$

3. Equation of a circle touching the line $|x-1|+|y-4|=6$ is
1) $x^2+y^2-2x-8y-1=0$
2) $x^2+y^2-2x-8y-18=0$
3) $x^2+y^2-2x-8y-17=0$
4) $x^2+y^2=4$

4. The coefficient of in the expansion of $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+...+1001x^{1000}$
1) $^{1000}C_{50}$ 2) $^{1001}C_{50}$
3) $^{1002}C_{50}$ 4) 2^{1001}

5. If $x=\sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then $y-x$ is equal to:
1) 10 2) 0
3) π 4) 7π

6. The minimum value of $|f(x)|$ where $f(x)$ is given by
$$f(x)=\frac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+\left(x^3+\frac{1}{x^3}\right)},$$
$x\neq 0$ is
1) 3 2) 6
3) 9 4) 12

7. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now from an A.P. Then the sum of the original three terms of the given G.P. is
1) 36 2) 24
3) 32 4) 28

8. The line $3x+2y=24$ meets x-axis at A and y-axis at B. The perpendicular bisector of $\overline{AB}$ meets the line passing through (0, –1) and parallel to x-axis at 'C'. Then the area of ΔABC is (in square units)
1) 83 2) 87
3) 90 4) 91

9. For a real number x, $[x]$ denotes the integral part of x. The value of $\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{100}\right]+\left[\frac{1}{2}+\frac{2}{100}\right]+...$
$..+\left[\frac{1}{2}+\frac{99}{100}\right]$ is
1) 49 2) 50
3) 48 4) 51

10. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is V then 200V is
1) $\frac{5}{2}$ 2) 1100
3) 6 4) $\frac{13}{2}$

11. If $x^2+y^2+z^2=r^2$, then
$\text{Tan}^{-1}\left(\frac{xy}{zr}\right)+\text{Tan}^{-1}\left(\frac{yz}{xr}\right)+\text{Tan}^{-1}\left(\frac{xz}{yr}\right)=$
1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$
3) $\pm\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

12. If normal to hyperbola $xy=c^2$ at $\left(ct_1,\frac{c}{t_1}\right)$ meet the curve again at $\left(ct_2,\frac{c}{t_2}\right)$, then:
1) $t_1t_2=-1$ 2) $t_2=-t_1-\frac{2}{t_1}$
3) $t_1t_1^3=-1$
4) $t_1^3t_2=-1$

13. If f is a real-valued differentiable function satisfying $|f(x)-f(y)|\le(x-y)^2$, $x, y\in R$ and $f(0)=0$, then $f(1)=$
1) 2 2) 0
3) –1 4) 5

14. Let three matrices $A=\begin{bmatrix}2&1\\4&1\end{bmatrix}$, $B=\begin{bmatrix}3&4\\2&3\end{bmatrix}$ and $C=\begin{bmatrix}3&-4\\-2&3\end{bmatrix}$ then
$\text{tr}(A)+\text{tr}\left(\frac{A(BC)}{2}\right)+\text{tr}\left(\frac{A(BC)^2}{4}\right)+\text{tr}\left(\frac{A(BC)^3}{8}\right)+.......\infty$ is equal to
1) 6 2) 14
3) 24 4) 18

15. The number of equivalence relations on a five element set is
1) 32 2) 42
3) 50 4) 52

16. If $|z_1|=2$, $|z_2|=3$, $|z_3|=4$ and $|z_1+z_2+z_3|=5$ then $|4z_2z_3+9z_3z_1+16z_1z_2|=$?
1) 120 b) 24
3) 48 4) 20

17. The number of symmetric matrices using –1, –1, –1, 1, 1, 1, 2, 2, 2 is
1) 24 2) 36
3) 48 4) 52

KEY & HINTS

1. 3;
Let $Z=a+ib$, $b\neq 0$ where
$\text{Im}Z=b$
$Z^5=(a+ib)^5=a^5+{}^5C_1a^4bi+{}^5C_2a^3b^2i^2+{}^5C_3a^2b^3i^3+{}^5C_4i^4+i^5b^5$
$\text{Im}Z^5=5a^4b-10a^2b^3+b^5$
$$y=\frac{\text{Im}Z^5}{\text{Im}^5Z}=5\left(\frac{a}{b}\right)^4-10\left(\frac{a}{b}\right)^2+1$$
Let $\left(\frac{a}{b}\right)^2=x(\text{say}), x\in R^+$
$y=5x^2-10x+1=5[x^2-2x]+1=5[(x-1)^2]-4$
Hence $y_{min}=-4$

2. 2;
$$=\underset{x\to 1^-}{\text{Lt}}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}\cdot\frac{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}$$
$$=\underset{x\to 1^-}{\text{Lt}}\frac{2\left(\frac{\pi}{2}-\sin^{-1}x\right)}{\sqrt{1-x}\left(\sqrt{\pi}+\sqrt{2\sin^{-1}x}\right)}$$
$$=\underset{x\to 1^-}{\text{Lt}}\frac{2\cos^{-1}x}{\sqrt{1-x}}\cdot\frac{2}{2\sqrt{\pi}}\quad \text{put } x=\cos\theta$$
$$=\underset{\theta\to 0^+}{\text{Lt}}\frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)}\cdot\frac{1}{2\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}$$

3. 1;
C =(1, 4), r = 3√2

4. 3;
Let,
$S=(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+...+1001x^{1000}$
$\frac{x}{1+x}S=x(1+x)^{999}+2x^2(1+x)^{998}+...+1000x^{1000}+\frac{1001x^{1001}}{1+x}$
Subtract above equations,
$\left(1-\frac{x}{1+x}\right)S=(1+x)^{1000}+(1+x)^{999}+x^2(1+x)^{998}+..+x^{1000}-\frac{1001x^{1001}}{1+x}$
$\Rightarrow S=(1+x)^{1001}+x(1+x)^{1000}+x^2(1+x)^{999}+....+x^{1000}(1+x)-1001]x^{1001}$
$$=\frac{(1+x)^{1001}\left[\left(\frac{x}{1+x}\right)^{1001}-1\right]}{\frac{x}{1+x}-1}-1001x^{1001}$$
[sum of G.P]
$=(1+x)^{1002}-x^{1002}-1002x^{1001}$
∴coefficient of x^{50} in S = coefficient of x^{50} in
$[(1+x)^{1002}-x^{1002}-1002x^{1001}]$
$={}^{1002}C_{50}$

5. 3;
$x=\sin^{-1}(\sin 10)=3\pi-10$
$y=\cos^{-1}(\cos 10)=4\pi-10$
$\Rightarrow y-x=\pi$

6. 2; Let ,
$a=\left(x+\frac{1}{x}\right)^3 \quad b=x^3+\frac{1}{x^3}$
$b^2=x^6+\frac{1}{x^6}+2$
$|f(x)|=\left|\frac{a^2-b^2}{a+b}\right|=|a-b|$
$=\left|\left(x+\frac{1}{x}\right)^3-x^3-\frac{1}{x^3}\right|$
$=\left|3\left(x+\frac{1}{x}\right)\right|\ge 3(2)=6$

7. 4;
Let terms are
$\frac{a}{r}, a, ar\to$ G.P
$\therefore a^3=512\Rightarrow a=8$
$\frac{8}{r}+4.12,8r\to$ A.P. $24=\frac{8}{r}+4+8r$
$r=2, r=\frac{1}{2}$ r =2 (4, 8, 16)
$r=\frac{1}{2}(16,8,4)$ Sum = 28

8. 4;
A(8, 0), B(0, 12). Midpoint of $\overline{AB}$ is (4, 6)
Equation to the perpendicular bisector of $\overline{AB}$ to $2x-3y+10=0$ —— (1)
The line through (0, –1) and parallel to x-axis is y = –1——(2)
Solving (1) and (2); C (–13/2, -1)
Area of
$$\Delta ABC=\frac{1}{2}\begin{vmatrix}8-0 & 0-12\\ 8+13/2 & 0+1\end{vmatrix}$$
$$=\frac{1}{2}\begin{vmatrix}8 & -12\\ 29/2 & 1\end{vmatrix}$$
$$=\frac{1}{2}\left|8\quad 1+12\quad \frac{29}{2}\right|=\frac{1}{2}\quad 182=91$$

9. 2;
∵ [x] denotes the integral part of x. Hence, after term $\left[\frac{1}{2}+\frac{50}{100}\right]$, each term will be one. Hence the sum of given series will be 50

10. 2;
$n_1=5, n_2=5, \sigma_1^2=4, \sigma_2^2=5$
$\bar{x}_1=2, \bar{x}_2=4\Rightarrow \bar{x}_{12}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}=3$
$d_1=(\bar{x}_1-\bar{x}_{12}), d_2=(\bar{x}_2-\bar{x}_{12})$
$\Rightarrow d_1=-1, d_2=1$
$$\sigma_{12} \text{ or } \sigma=\sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2+n_1d_1^2+n_2d_2^2}{n_1+n_2}}$$
$$=\sqrt{\frac{11}{2}}\Rightarrow\sigma_{12}^2=\frac{11}{2}\quad 200=1100$$

11. 4;
Put $x=y=z=1$
$\Rightarrow r^2=3\Rightarrow r=\sqrt{3}$
$\Rightarrow 3\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{2}$

12. 4;
Hyperbola Equation of normal at $\left(ct_1,\frac{c}{t_1}\right)$ is
$t_1^3x-t_1y-ct_1^4+c=0$
It passes through $\left(ct_2,\frac{c}{t_2}\right)$
i.e $\left(t_1^3.ct_2-t_1\frac{c}{t_2}-ct_1^4+c=0\right)$
$\Rightarrow(t_1-t_2)(t_1^3t_2+1)=0$
$\Rightarrow t_1^3t_2=-1$

13. 2;
$$\underset{x\to y}{\text{Lim}}\frac{|f(x)-f(x)|}{|x-y|}\le\underset{x\to y}{\text{Lim}}|x-y|$$
$\Rightarrow|f^1(y)|\le 0\Rightarrow f^1(y)=0$
$\Rightarrow f(y)=\text{constant as } f(0)=0$
$\Rightarrow f(y)=0\Rightarrow f(1)=0$

14. 1;
$=1+\frac{1}{3}+\frac{1}{5}+......+\frac{1}{17}$
$BC=\begin{bmatrix}3&4\\2&3\end{bmatrix}\begin{bmatrix}3&-4\\-2&3\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$
then
$s=\text{tr}(A)+\text{tr}\left(\frac{A}{2}\right)+\text{tr}\left(\frac{A}{2^2}\right)+....$
$...=\text{tr}(A)+\frac{1}{2}\text{tr}(A)+\frac{1}{2^2}\text{tr}(A)+....$
$=\text{tr}(A)\left[\frac{1}{1-½}\right]=\text{tr}(A)(2)$
$=2\text{tr}(A)=2(3)=6$

15. 4;
Note that equivalence relations and partitions are same in number.
$=\sum_{r=0}^{4}\binom{4}{r}P_r=\binom{4}{0}P_0+\binom{4}{1}P_1+\binom{4}{2}P_2+\binom{4}{3}P_3+\binom{4}{4}P_4$
Now,
$P_0=1, P_1=1, P_2=\binom{1}{0}P_0+\binom{1}{1}P_1=1+1=2$
$P_3=\binom{2}{0}P_0+\binom{2}{1}P_1+\binom{2}{2}P_2=1+2.1+1.2=5$
$P_4=\binom{3}{0}P_0+\binom{3}{1}P_1+\binom{3}{2}P_2+\binom{3}{3}P_3$
$=1.1+3.1+3.2+1.5=15$
$P_5=\binom{4}{0}P_0+\binom{4}{1}P_1+\binom{4}{2}P_2+\binom{4}{3}P_3+\binom{4}{4}P_4$
$=1.1+4.1+6.2+4.5+1.15$
$=1+4+12+20+15=52$

16. 1;
$|4z_2z_3+9z_3z_1+16z_1z_2|$
$=|z_1\bar{z}_1z_2z_3+\bar{z}_2z_2z_3z_1+\bar{z}_3z_3z_1z_2|$
$=|z_1||z_2||z_3||z_1+z_2+z_3|=120$

17. 2;
Req.no.of ways
= 3! × 3!
= 36