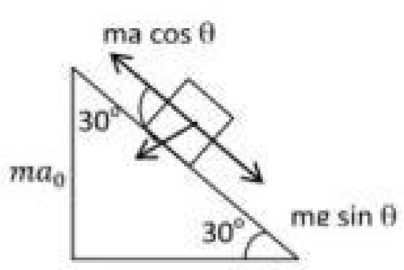
Which is called kinetic friction?

NEWTONS LAWS OF MOTION, FRICTION & UNIFORM CIRCULAR MOTION

Continued from November 12th..

W.E - 24: A block is placed on an inclined plane moving towards right with an acceleration $a_0 = g$. The length of the inclined plane is l_0. All the surfaces are smooth. Find the time taken by the block to reach from bottom to top.



Sol: ma = ma_0cos30° – $mgsin30^\circ$

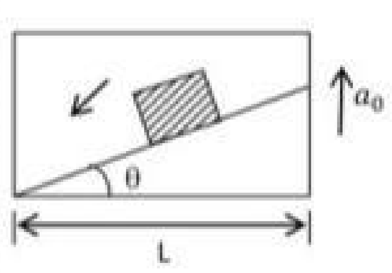
$a = \frac{ma_0cos30^0 - mgsin30^0}{m}$

$a = \frac{ma_0\frac{\sqrt{3}}{2} - mg\frac{1}{2}}{m} = g\left(\frac{\sqrt{3}-1}{2}\right)$

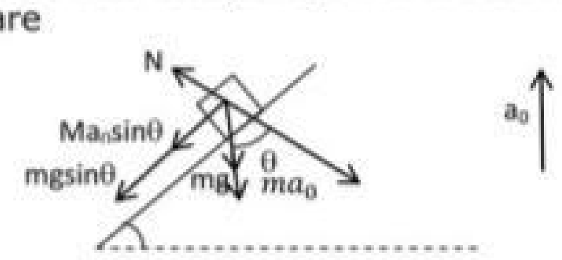
From $s = +\frac{1}{2}at^2$; $l_0 = \frac{1}{2}at^2$

$l_0 = \frac{1}{2}g\left(\frac{\sqrt{3}-1}{2}\right)t^2 \Rightarrow t = \sqrt{\frac{4l_0}{g(\sqrt{3}-1)}}\,sec$

W.E - 25: A block slides down from top of a smooth inclined plane of elevation. Fixed in an elevator going up with an acceleration a_0. The base of incline has length L. Find the time taken by the block to reach the bottom.



Sol: Let us solve the problem in the elevation frame. The free body diagrams is shown. The forces are



i) N normal reaction to the plane,
ii) mg acting vertically downwards,
iii) ma_0 (Pseudo force). Acting vertically down

If a is acceleration of the body with respect to inclined plane, taking components of forces parallel to the inclined plane.

$mgsin\theta + ma_0sin\theta = ma$

$\therefore a = (g + a_0)sin\theta$

This is the acceleration with respect to the elevator

The distance travelled is $\frac{L}{cos\theta}$. If 't' is the time for reaching the bottom of inclined plane

$\frac{L}{cos\theta} = 0 + \frac{1}{2}(g + a_0)sin\theta t^2$

$t = \left[\frac{2L}{(g+a_0)sin\theta cos\theta}\right]^{\frac{1}{2}} = \left[\frac{4L}{(g+a_0)sin2\theta}\right]^{\frac{1}{2}}$

Law of conservation of momentum:

- When the resultant external force acting on a system is zero, the total momentum (vector sum) of the system remains constant. This is called "law of conservation of linear momentum".
- Newton's third law of motion leads to the law of conservation of linear momentum.
- Walking ,running, swimming, jet propulsion, motion of rockets, rowing of a boat, recoil of a gun etc., can be explained by Newton's third law of motion.
- Explosions, disintegration of nuclei, recoil of gun collisions etc., can be explained on the basis of the law of conservation of linear momentum.

Applications:

- When a shot is fired from a gun, while the shot moves forwards, the gun moves backwards. This motion of gun is called recoil of the gun. When a gun of mass 'M' fires a bullet of mass 'm' with a muzzle velocity 'v' (relative velocity of bullet w.r.t gun), the gun recoils with a velocity 'v' given by V=mv/M.
- When a bullet of mass 'm' moving with a velocity 'v' gets embedded into a block of mass M at rest and free to move on a smooth horizontal surface, then their common velocity V=mv/(M+m).
- A boy of mass 'm' walks a distance 's' on a boat of mass 'M' that is floating on water and initially at rest. If the boat is free to move, it moves back a distance d =ms/(M+m).

Explosion of Bomb

- A shell of mass 'M' at rest explodes into two fragments and one of masses 'm' moves out with a velocity 'v' the other piece of mass (M-m) moves in opposite direction with a velocity of V=mv/(M-m).
- Suppose a shell of mass m at rest explodes into three pieces of masses m_1,m_2 and m_3, moving with velocities $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ respectively.

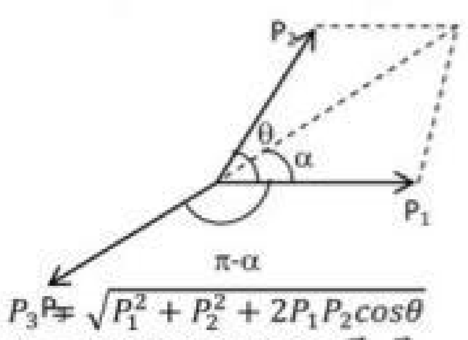
$m_1\vec{v}_1 = \vec{p}_1$; $m_2\vec{v}_2 = \vec{p}_2$; $m_3\vec{v}_3 = \vec{p}_3$

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$

(as shell is at rest initially)

$\therefore \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

So the third piece moves with the same magnitude of the resultant momentum of the other two pieces but in opposite direction.



$P_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2cos\theta}$

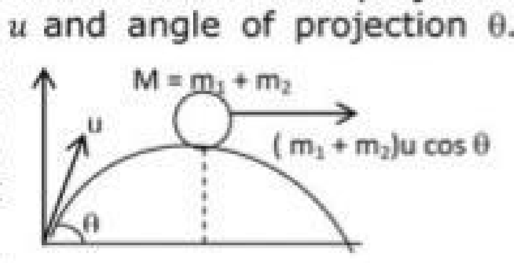
θ = angle between $\vec{P}_1, \vec{P}_2$

$(\pi - \alpha)$ = angle between $\vec{P}_3, \vec{P}_1$

$tan\alpha = \frac{P_2sin\theta}{P_1+P_2cos\theta}$

Explosion of a shell travelling in a parabolic path at its highest point: (into two fragments)

- Consider a shell of mass M as a projectile with velocity u and angle of projection θ. Suppose the shell breaks into two fragments at maximum height and their initial velocities are $\vec{v}_1$ and $\vec{v}_2$



Total momentum of the two parts is constant just before and just after the explosion.

$[m_1 + m_2]ucos\theta\hat{i} = m_1\vec{v}_1 + m_2\vec{v}_2$

Case: (i) If the fragments travel in opposite direction after explosion then

$(m_1 + m_2)ucos\theta\hat{i} = m_1v_1\hat{i} - m_2v_2\hat{i}$

Case : (ii) If one fragment retraces its path and falls at the point of projection

$(m_1 + m_2)ucos\theta\hat{i} = -m_1ucos\theta\hat{i} + m_2\vec{v}_2$

Case : (iii) If one fragment falls freely after explosion $(m_1 + m_2)ucos\theta\hat{i} = m_10 + m_2\vec{v}_2$

$(m_1 + m_2)ucos\theta\hat{i} = m_2\vec{v}_2$

W.E - 26: A bomb moving with velocity (40i+50j-25k)m/s explodes into two pieces of mass ratio 1:4. After explosion the smaller piece moves away with velocity (200i+70j+15k)m/s. The velocity of larger piece after explosion is

Sol: From Law of conservation of linear momentum

$Mu = m_1v_1 + m_2v_2$; M = 5x, m_1 = x, m_2=4x

$u = 40\hat{i} + 50\hat{j} - 25\hat{k}ms^{-1}$;

$v_1 = (200\hat{i} + 70\hat{j} + 15\hat{k})ms^{-1}$

Here v_2 is the velocity of the larger piece

$5x(40\hat{i} + 50\hat{j} - 25\hat{k}) = xv_1 = (200\hat{i} + 70\hat{j} + 15\hat{k}) + 4x(v_2)$

On simplification, we get $v_2 = 45\hat{j} - 35\hat{k}$

W.E – 27: A particle of mass 4 m explodes into three pieces of masses m, m and 2m. The equal masses move along X-axis and Y-axis with velocities $4ms^{-1}$ and $6ms^{-1}$ respectively. The magnitude of the velocity of the heavier mass is

Sol: M =4m, u=0,m_1=m.m_2=m.m_3=2m

$V_1 = 4ms^{-1}$. $V_2 = 6ms^{-1}$. V_3=?

According to law of conservation of momentum,

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$

$\vec{P}_3 = -(\vec{P}_1 + \vec{P}_2), |\vec{P}_3| = |\vec{P}_1 + \vec{P}_2|$

$p_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2Cos\theta}$

P_1 and P_2 are perpendicular to each other

$p_3 = \sqrt{P_1^2 + P_2^2}$, $m_2v_2 = \sqrt{(m_1v_1)^2 + (m_2v_2)^2}$

$2mv_3 = \sqrt{(m \times 4)^2 + (m \times 6)^2}$

$2v_3 = \sqrt{16 + 36} \Rightarrow v_3 = \sqrt{13}ms^{-1}$

W.E - 28: A rifle of 20kg mass can fire 4 bullets/s. The mass of each bullet is $30x10^{-3}$ kg and its final velocity is $400ms^{-1}$. Then, what force must be applied on the rifle so that it does not move backwards while firing the bullets?

Sol: Law of conservation of momentum

$MV + 4mv = 0 \Rightarrow V = \frac{4mv}{M} = \frac{4 \times 35 \times 10^{-3} \times 400}{20} = -2.8ms^{-1}$

Force applied on the rifle

$F = \frac{MV}{t} = -\frac{20 \times 2.8}{1} = -56N$

W.E – 29: A bomb of 1 kg is thrown vertically up with speed 100 m/s. After 5 seconds, it explodes into two parts. One part of mass 400 gm does down with speed 25 m/s. What will happen to the other part just after explosion

Sol: After 5 sec, velocity of the bomb,

$v = u + at$

$\vec{v} = u\hat{j} - gt\hat{j} = (100 - 10 \times 5)\hat{j} = 50\hat{j}m/s$

m=1kg, m_1=0,4kg, m_2=0.6kg,v_1=$25ms^{-1}$

According to law of conservation of momentum

$m\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$

$1 \times 50\hat{j} = -0.4 \times 25\hat{j} + 0.6\vec{v}_2$

$\Rightarrow v_2 = 100\hat{j} = v_2 = 100ms^{-1}$,

vertically upwards

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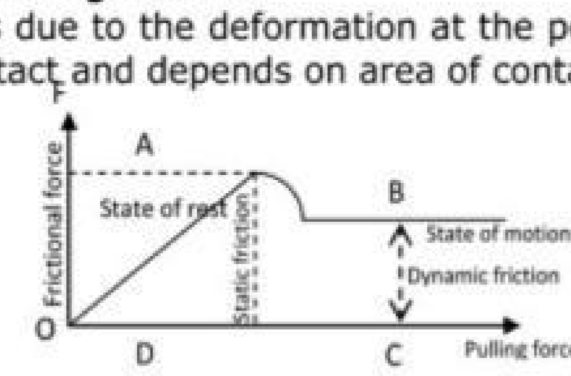
Friction: If we slide or try to slide a body over another surface, the motion of the body is resisted by bonding between the body and the surface. This resistance is called Friction

- The force of friction is parallel to the contact surfaces and opposite to the direction of intended or relative motion
- There are three types of frictional forces

(i) Static friction (iii) Rolling friction
(ii)Dynamic friction

- If a body is at rest and no pulling force is acting on it, force of friction on it is zero
- If a force is applied to move the bodyand it does not move, the friction developed is called **static friction**, which is equal in magnitude and opposite in direction to the applied force (static friction is self adjusting force)
- If a force is applied to move the body and it moves, then the friction developed is called **dynamic or kinetic friction**
- When a body rolls on the surface of another body friction developed is called as **rolling friction**
- It is due to the deformation at the point of contact and depends on area of contact.



Note-I: If you are walking due east, then the friction on the feet is due east and the friction on the surface is due west

Note-II: Engine is connected to rear wheels of a car. When the car is accelerated, direction of frictional force on the rear wheels will be in the direction of motion and on the front wheels in the opposite direction of motion

Note-III: In cycling, the force exerted by rear wheel on the ground makes the force of friction to act on it in the forward direction. Front wheel moving by itself experience force of friction in backward direction

Note-iv: If the pedaling cycle is accelerating on the horizontal surface, then the forward friction on the rear wheel is greater than the backward friction on the front wheel

Note-v: When pedaling is stopped, the frictional force is in backward direction for both the wheels.

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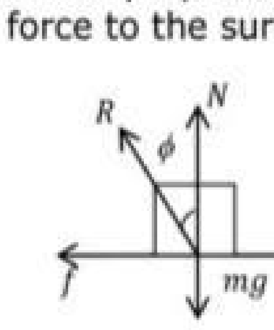
Laws of Friction:

- Limiting friction is directly proportional to the normal reacting acting on the body
- The law of static friction may thus be written as $f_s \alpha N. \Rightarrow (f_s)_{max} = \mu_s N = f_1$ Generally 0 ≤ static friction ≤ f_1 Where the dimensionless constant μ_s is called the coefficient of static friction and N is the magnitude of the normal force $(f_1)_{max} = f_1 = \mu N; f_1$ Limiting friction
- Coefficient of static friction (μ_s) depends on the nature of the two surfaces in contact and is independent of the area of contact
- Static friction is independent of the area of contact between the two surfaces
- Coefficient of kinetic friction $(\mu_x) = \frac{f_1}{N}$ It is independent of velocity of the body
- Coefficient of rolling friction $(\mu_x) = \frac{f_R}{N}$
- Rolling friction depends on the area of the surfaces in contact Note: $\mu_s > \mu_K > \mu_R$
- Friction depends on the nature of the two surfaces in contact i.e., nature of materials, surface finish, temperature of the two surfaces etc.

Angle of friction:

- Angle made by the resultant of f and N with the normal reaction N is called angle of friction
- Friction is parallel component of contact force to the surfaces
- Normal force is perpendicular component of contact force to the surfaces



$R = \sqrt{f^2 + N^2}$

- When the block is static $tan\phi = \frac{f}{N}; \phi \leq \phi_1$
- When the block is in impending state, $\phi_1 = \frac{u_2N}{N} = \mu_1$ Where $\phi_1 \rightarrow$ maximum angle of friction
- When block is sliding, $tan\phi_1 = \frac{\mu_1N}{N} = \mu_1$ Since $\mu_1 > \mu_2$, it follows that $\phi_l > \phi_k$

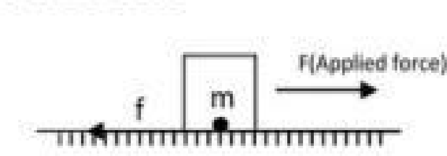
$F_R = \sqrt{f_1^2 + N^2} = \sqrt{(\mu_1N)^2 + N^2}$ =N$\sqrt{\mu_1^2 + 1}$

$F_B = mg\sqrt{tan^2\phi_1 + 1}$ $(\because \mu_1 = tan\phi_1)$

$F_B = mgsec\phi_1$

Motion on a horizontal rough surface: Consider a block of mass 'm' placed on a horizontal surface with normal reaction N.

Case (i): If applied force F=0, then the force of friction is also zero



Case (ii): If applied force F < $(f_s)_{max}$,the block does not move and the force of friction is $f_s = F$

Find the common difference?

TRIGONOMETRIC EQUATIONS

4 Marks: -

1) If α is the Principal Value (P.V) of θ satisfying $sin\theta = k$ then $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2) If α is the Principal Value (P.V) of θ satisfying $\cos\theta = k$ then $\alpha \in [0, \pi]$

3) If α is the Principal Value (P.V) of θ satisfying $\cos\theta = k$ then $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

4) Basic Trigonometric equations and their General Solutions:

Basic trigonometric equation	General Solution (G.S)
1) $\sin\theta = 0$	$\theta = n\pi, n \in Z$
2) $\cos\theta = 0$	$\theta = (2n+1)\frac{\pi}{2}, n \in Z$
3) $\tan\theta = 0$	$\theta = n\pi, n \in Z$
4) $\sin\theta = k = \sin\alpha, -1 \le k \le 1, \alpha \in [-\pi/2, \pi/2]$	$\theta = n\pi + (-1)^n\alpha, n \in Z$
5) $\cos\theta = k = \cos\alpha, -1 \le k \le 1, \alpha \in [0, \pi]$	$\theta = 2n\pi \pm \alpha, n \in Z$
6) $\tan\theta = k = \tan\alpha, k \in R, \alpha \in (-\pi/2, \pi/2)$	$\theta = n\pi + \alpha, n \in Z$
7) $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha$. Where α is corres-ponding principal value	$\theta = n\pi \pm \alpha, n \in Z$
8) In simultaneous trigon-ometric equations, the principal value 'α' satisfying both the given equations can be obtained by locating the quadrant in which α lies (observe the signs of the given trigonometric functions)	$\theta = 2n\pi + \alpha, n \in Z$

Short Answers & Questions

1) Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$

Sol: Given equation is $\sin x + \sqrt{3}\cos x = \sqrt{2}$ on dividing by $\sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$, we get $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \Rightarrow \sin 30^o \sin x + \cos 30^o \cos x = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos x\cos\frac{\pi}{6} + \sin x\sin\frac{\pi}{6} + \sin x\sin\frac{\pi}{6} = \cos\frac{\pi}{4} \Rightarrow$

$\cos\left(x - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}$, here P.V is $\alpha = \frac{\pi}{4}$

$\therefore$ General solution is given by $\theta = 2n\pi \pm \alpha, n \in Z$

$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4} \Rightarrow 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}, n \in Z$

$\Rightarrow$**Solve $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$ $\left[Ans: \theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{6}, n \in Z\right]$**

2) Solve $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$

Sol: Given equation is $\sqrt{2}(\sin x + \cos x) = \sqrt{3} \Rightarrow \sqrt{2}\sin x + \sqrt{2}\cos x = \sqrt{3}$

On dividing by $\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2+2} = \sqrt{4} = 2$,

we get $\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = \frac{\sqrt{3}}{2} \Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{\sqrt{3}}{2} \Rightarrow \cos x\cos\frac{\pi}{4} + \sin x\sin\frac{\pi}{4} = \cos\frac{\pi}{6}$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{6}$ (Here P.V is $\alpha = \frac{\pi}{6}$)

$\therefore$ General solution is given by $\theta = 2n\pi \pm \alpha$, $n \in Z$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{6} \Rightarrow x = 2n\pi \pm \frac{\pi}{6} + \frac{\pi}{4}, n \in Z$

3) Solve cotx+cosecx=$\sqrt{3}$

Sol: cotx+cosecx=$\sqrt{3} \Rightarrow \frac{\cos x}{\sin x} + \frac{1}{\sin x} = \sqrt{3} \Rightarrow \cos x + 1 = \sqrt{3}\sin x \Rightarrow \sqrt{3}\sin x - \cos x = 1$

Dividing by $\sqrt{3+1} = 2$, we have

$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{1}{2} \Rightarrow \cos\frac{\pi}{6}\sin x - \sin\frac{\pi}{6}\cos x = \frac{1}{2} \Rightarrow \sin\left[x - \frac{\pi}{6}\right] = \sin\frac{\pi}{6}$

$\therefore$ The general solution is $x - \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{6} \Rightarrow x = n\pi + (-1)^n\frac{\pi}{6} + \frac{\pi}{6}, n \in Z$

4) Solve 1 + sin2x = (sin3x - cos3x)^2

Sol: 1 + sin2x = (sin3x - cos3x)$^2 \Rightarrow$

1 + sin2x = sin^{2}3x + cos^{2}3x – 2sin3x cos3x

$\Rightarrow$1+sin2x=1-sin2(3x)$\Rightarrow$sin6x+sin2x=0

$\Rightarrow 2\sin\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right) = 0$

$\Rightarrow \sin(4x).\cos(2x) = 0 \Rightarrow$sin4x = 0 (or) cos2x = 0

(i) If sin4x = 0 then 4x = n$\pi \Rightarrow x = \frac{n\pi}{4}, n \in Z$

(ii) If cos2x = 0 then 2x = $(2n+1)\frac{\pi}{2}, n \in Z$

$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{4}, n \in Z$

The general solution is $x = \frac{n\pi}{4}; \frac{n\pi}{2} + \frac{\pi}{4}, n \in Z$

5) Solve 1 + sin$^2\theta$ = 3 sinθ cosθ

Sol: Dividing the given equation, both sides by cos$^2\theta$, we get $\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cos\theta}{\cos^2\theta}, \cos\theta \ne 0$

$\Rightarrow$ sec$^2\theta$ + tan$^2\theta$= 3tan$\theta \Rightarrow$ (1+tan$^2\theta$) +tan$^2\theta$= 3 tan$\theta \Rightarrow$ 2 tan$^2\theta$ – 3 tanθ +1 = 0

$\Rightarrow$ (2tanθ - 1) (tan θ -1) = 0 $\Rightarrow$ tanθ = 1

(or) tan θ = ½

Now, tan θ = 1 =tan $^{\pi}/_4 \Rightarrow$ Principal solution is $\alpha = ^{\pi}/_4$

$\therefore$ The general solution is $\theta = n\pi + \frac{\pi}{4}, n \in Z$ Also, tan$\theta = \frac{1}{2} \Rightarrow \theta = \text{Tan}^{-1}\frac{1}{2}$

$\therefore$ Principal solutions is $\alpha = \text{Tan}^{-1}\frac{1}{2}$

$\therefore$ The general solution is $\theta = n\pi + \text{Tan}^{-1}\frac{1}{2}, n \in Z$

6) Solve tanθ + 3cotθ = 5 secθ.

Sol: The given equation is tanθ + 3cotθ = 5secθ

$\Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{3\cos\theta}{\sin\theta} = \frac{5}{\cos\theta}$,

(which is valid when cos$\theta \ne$0,sin$\theta \ne$0)

$\Rightarrow \frac{\sin^2\theta + 3\cos^2\theta}{\sin\theta\cos\theta} = \frac{5}{\cos\theta} \Rightarrow \sin^2\theta + 3\cos^2\theta = 5\sin\theta$

$\Rightarrow \sin^2\theta + 3(1 - \sin^2\theta) = 5\sin\theta$

$\Rightarrow \sin^2\theta + 3 - 3\sin^2\theta = 5\sin\theta$

$\Rightarrow 2\sin^2\theta + 5\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 3) = 0$

$\Rightarrow$ sinθ = 1/2 (Here, sinθ = - 3 has no solution)

$\therefore$ Principal solution is $\alpha = \pi/6$

Hence, the general solution is $\theta = n\pi + (-1)^n\frac{\pi}{6}$,n$\in Z$

7) If acos2θ + bsin2θ = c has θ_1, θ_2 as its solutions then show that $tan\theta_1 + tan\theta_2 = \frac{2b}{c+a}$ $tan\theta_1 \cdot tan\theta_2 = \frac{c-a}{c+a}$ and hence show that tan $(\theta_1 + \theta_2)$=b/a.

A) Given equation is a cos2θ + b sin2θ=c

$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$

$\Rightarrow$ a(1 - tan$^2\theta$) + b(2tanθ) = c(1 + tan$^2\theta$)

$\Rightarrow$a – a tan$^2\theta$ + 2b tanθ = c + c tan$^2\theta$

$\Rightarrow$ ca tan$^2\theta$ + a tan$^2\theta$ + 2b tanθ + (c - a) = 0 $\Rightarrow$(a + c) tan$^2\theta$ + (c - a) = 0 ------ (1)

(1) is a quadratic equation in tanθ

But θ_1, θ_2 are the solutions of $\theta \Rightarrow$ tanθ_1,tan$\theta_2$are the roots of equation (1)

$\therefore$sum of the roots = tanθ_1+ tanθ_2 = $\frac{2b}{a+c}$

Product of the roots = tan$\theta_1 \cdot$ tanθ_2=$\frac{c-a}{c+a}$

$\therefore \tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2} = \frac{\frac{2b}{c+a}}{1 - \left(\frac{c-a}{c+a}\right)} = \frac{\frac{2b}{c+a}}{\frac{c+a-c+a}{c+a}}$

$= \frac{2b}{2a} = \frac{b}{a}$

8)If α, β are the solutions of the equation acosθ + b sinθ = c, where a, b, c$\in$ R and if a^2 + b^2 > 0 , cos $\alpha \ne$ cos β and sin $\alpha \ne$ sin β, then S.T

(i) sinα+sinβ = $\frac{2bc}{a^2+b^2}$

(ii) sin$\alpha \cdot$sinβ=$\frac{c^2-b^2}{a^2+b^2}$

Sol: Given equation is acosθ+bsinθ=c

$\Rightarrow$acosθ = c – bsinθ.

By squaring on both sides, we get a^{2}cos$^2\theta$ = (c - bsinθ)^2

$\Rightarrow a^2 \cdot (1 - \sin^2\theta) = c^2 + b^2\sin^2\theta - 2bc\sin\theta$

$\Rightarrow (a^2 + b^2)\sin^2\theta - 2bc\sin\theta + (c^2 - a^2)$

= 0 …… (1)

(1) is a quadratic equation in sinθ.

------ α,β are solutions for the given equation, sinα and sinβ are the roots of (1)

Sum of the roots = sinα + sinβ = $\frac{2bc}{a^2+b^2}$; Product of the roots = sinα.sinβ = $\frac{c^2-a^2}{a^2+b^2}$

If α , β are the solutions of the equation acosθ+bsinθ=c, where $a, b, c \in R$ and if $a^2 + b^2 >$ 0

cos$\alpha \ne$cosβ, sin$\alpha \ne$sinβ, then show that (i) cosα+cosβ = $\frac{2ac}{a^2+b^2}$ (ii) cos$\alpha \cdot$cosβ=$\frac{c^2-b^2}{a^2+b^2}$

9) Given p$\ne \pm$q, show that the solutions of cospθ+cosqθ = 0 form two series each of which is in A.P. Find also the common difference of each A.P.

Sol: Given cospθ+cosqθ = 0

$\Rightarrow 2\cos\left[\left(\frac{p+q}{2}\right)\theta\right]\cos\left[\left(\frac{p-q}{2}\right)\theta\right] = 0$

$\Rightarrow \cos\left(\frac{p+q}{2}\right)\theta = 0$ (or) $\cos\left(\frac{p-q}{2}\right)\theta = 0$

(i) $\cos\left(\frac{p+q}{2}\right)\theta = 0 = \cos\frac{\pi}{2}$

$\Leftrightarrow \left(\frac{p+q}{2}\right)\theta = 2n\pi \pm \frac{\pi}{2}(4n \pm 1)\frac{\pi}{2} \Leftrightarrow \theta = \frac{(4n \pm 1)}{(p+q)}, n \in Z$

The solutions are: ……….., $-\frac{\pi}{p+q}, \frac{\pi}{p+q}, \frac{3\pi}{p+q}, \frac{5\pi}{p+q}$, …. …

These solutions form an A.P with common difference $\frac{2\pi}{(p+q)}$

(ii) $\cos\left(\frac{p-q}{2}\right)\theta = 0\cos\frac{\pi}{2}$

$\Leftrightarrow \left(\frac{p-q}{2}\right)\theta = 2n\pi \pm \frac{\pi}{2} = (4n \pm 1)\frac{\pi}{2} \Leftrightarrow \theta = \frac{(4n \pm 1)\pi}{(p-q)}, n \in Z$

The solutions are: ……….., $-\frac{\pi}{p-q}, \frac{\pi}{p-q}, \frac{3\pi}{p-q}, \frac{5\pi}{p-q}$, … …

These solutions form an A.P with common difference $\frac{2\pi}{(p-q)}$

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10) Show that the solutions of cospθ=sinqθ form two series each of which is an A.P. Find also the common difference of each A.P. (P$\ne \pm$q)

11) If tanpθ=cosqθ and p$\ne$-q show that the solutions are in A.P with common difference $\frac{\pi}{p+q}$

12) Find all values of x in $-\pi, \pi)$ satisfying the equation $8^{1+\cos x+\cos^2 x+}$ ….. = 4^3.

Sol: $8^{1+\cos x+\cos^2 x+}$ ….. = 4^3 = 64 = 8^2

$\Rightarrow$1+cosx+cos^{2}x+….= 2

Now, 1 + cosx+cos^{2}x+…= $\frac{1}{1-\cos x}$ $\left(since|\cos x| < 1, S_\infty = \frac{a}{1-r}\right)$

$\therefore \frac{1}{1-\cos x} = 2 \Rightarrow 1 - \cos x = \frac{1}{2} \Rightarrow \cos x = 1 - \frac{1}{2} = \frac{1}{2} = \cos\left(\pm\frac{\pi}{3}\right) \Rightarrow x = \pm\frac{\pi}{3}$ $[\because x \in (-\pi, \pi)]$

HYPERBOLIC FUNCTIONS

2 Marks Synopsis Points

Hyperbolic function	Definition	Domain	Range
1. sinhx	$\frac{e^x - e^{-x}}{2}$	R	R
2.coshx	$\frac{e^x + e^{-x}}{2}$	R	$(1,\infty)$
3.tanhx	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	R	(-1,1)
4.cothx	$\frac{e^x + e^{-x}}{e^x - e^{-x}}$	R – {0}	$(-\infty,1)\cup(1,\infty)$
5.sec hx	$\frac{2}{e^x + e^{-x}}$	R	(0,1]
6.cschx	$\frac{2}{e^x - e^{-x}}$	R – {0}	R – {0}
7.$\sinh^{-1}x$	$\log\left(x + \sqrt{x^2+1}\right)$	R	R
8.$\text{Cosh}^{-1}x$	$\log\left(x + \sqrt{x^2-1}\right)$	$[1,\infty)$	$[0,\infty)$
9.$\text{Tanh}^{-1}x$	$\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$	(1–,1)	R
10.$\text{Coth}^{-1}x$	$\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$	$(-\infty,1)\cup(1,\infty)$	R – {0}
11.$\text{Sech}^{-1}x$	$\log\left(\frac{1+\sqrt{1-x^2}}{x}\right)$	(0,1]	$[0,\infty)$
12.$\text{Csch}^{-1}x$	$\log\left(\frac{1 \pm \sqrt{1+x^2}}{x}\right)$	R – {0}	R – {0}

Hyperbolic identities Verses	Trigonometric identities
13. sinh2x=2sinhxcoshx	sin2θ=2sinθcosθ
14.cosh2x=cosh^{2}x+sinh^{2}x	cos2θ=cos$^2\theta$-sin$^2\theta$
15.cosh^{2}x-sinh^{2}x=1	cos$^2\theta$+sin$^2\theta$=1
16.sech^{2}x +tanh^{2}x=1	sec$^2\theta$ - tan$^2\theta$=1
17.coth^{2}x – csch^{2}x =1	csc$^2\theta$ - cot$^2\theta$ = 1

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VSAQ (1×2=2)

1) Prove that cosh^{2}x – sinh^{2}x =1

Sol: L.H.S cosh^{2}x – sinh^{2}x

$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \frac{1}{4}(4.e^x.e^{-x}) = 1 =$ R. H. S

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

SPQ - P.T cosh^{2}x + sinh^{2}x = cosh2x

2) Prove that cosh^{4}x – sinh^{4}x = cosh2x

Sol: cosh^{4}x – sinh^4

= (cosh^{2}x – sinh^{2}x)(cosh^{2}x + sinh^{2}x)

= (1) (cosh2x) = cosh2x

3) Prove that sinh(3x) = 3sinhx + 4sinh^{3}x,$\forall x \in$ R

Sol: L.H.S = sinh3x = sinh(2x+x)

= sinh2x·coshx+cosh2x·sinhx

=(2sinhx·coshx)coshx +(1+2sinh^{2}x)sinhx

=2sinhx(cosh^{2}x)+(1+2sinh^{2}x)sinhx

= 2sinhx(1+sinh^{2}x)+(1+2sinh^{2}x)sinhx

=2sinhx+2sinh^{3}x+sinhx+2sinh^{3}x

= 3sinhx+4sinh^{3}x= R.H.S

SPQ – Prove that cosh3x = 4cosh^3 – 3coshx

4) P.T tanh3x = $\frac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$, $\forall x \in R$

Sol: L.H.S = tan3x = tanh(2x + x)

$= \frac{\tanh 2x + \tanh x}{1 + \tanh 2x \tanh x}$

$= \frac{\frac{2\tanh x}{1+\tanh^2 x} \times \tanh x}{1 + \left(\frac{2\tanh x}{1+\tanh^2 x}\right)\tan x}$

$= \frac{2\tanh x + \tanh x(1 + \tanh^2 x)}{1 + \tanh^2 x + 2\tanh^2(x)}$

$= \frac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$ = R.H.S

5) Prove that (coshx – sinhx)^n = cosh(nx) – sinh(nx)

Sol: L.H.S = (coshx – sinhx)^n

$= \left[\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2}\right]^n$

$= \left[\frac{e^x + e^{-x} - e^x + e^{-x}}{2}\right]^n$

$= \left(\frac{2e^{-x}}{2}\right)^n = e^{-nx}$

R.H.S = cosh nx – sinh nx

$= \left(\frac{e^{nx} + e^{-nx}}{2}\right) - \left(\frac{e^{nx} - e^{-nx}}{2}\right)$

$= \frac{e^{nx} + e^{-nx} - e^{nx} + e^{-nx}}{2}$

$= \frac{2e^{-nx}}{2} = e^{-nx}$

$\therefore (\cosh x - \sinh x)^n = \cosh nx - \sinh nx$

SPQ - Prove that $(\cosh x + \sinh x)^n = \cosh(nx) + \sinh(nx)$