The difference between the accelerations of ...

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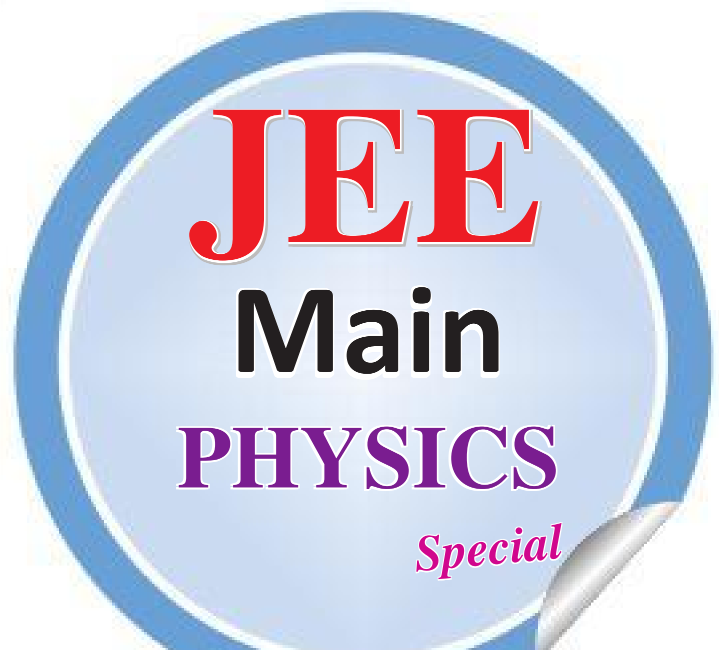
Dr. RK's Classes

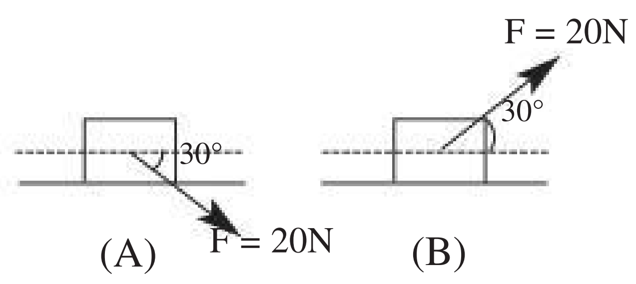


MODEL QUESTIONS

1. The force of interaction between two atoms is given by F αβexp$\left(-\frac{x^2}{\alpha kt}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is :
1) $M^2L^2T^{-2}$ 2) M^2LT^{-4}
3) $M^0L^2T^{-4}$ 4) MLT^{-2}

2. A particle is moving with speed v=b√x along positive x-axis. Calculate the speed of the particle at time t = τ (assume that the particle is at origin at t = 0)
1) $\frac{b^2\tau}{4}$ 2) $\frac{b^2\tau}{2}$
3) $b^2\tau$ 4) $\frac{b^2\tau}{\sqrt{2}}$

3. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20 N, making an angle of 30º with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is   = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be : (g = 10 ms^{-2})

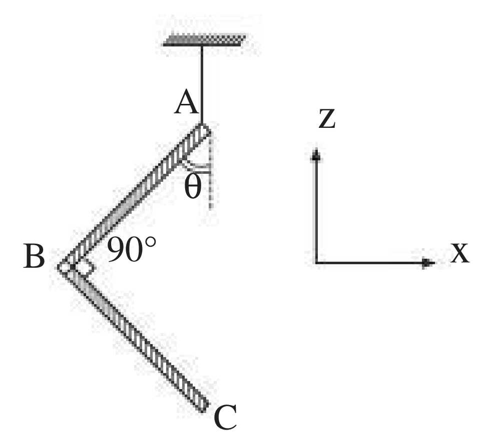


1) 0 ms^{-2} 2) 0.8 ms^{-2}
3) 0.4 ms^{-2} 4) 3.2 ms^{-2}

4. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma\upsilon^2$ (where m is mass of the ball, υ is its Instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is:
1) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$
2) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$
3) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$
4) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$

5. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l(l << L), is close to :
1) Mgl 2) $Mgl(1+\theta_0^2)$
3) $Mgl(1-\theta_0^2)$ 4) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$

6. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ , then

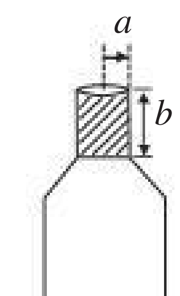


1) $\tan\theta=\frac{2}{\sqrt{3}}$ 2) $\tan\theta=\frac{1}{3}$
3) $\tan\theta=\frac{1}{2}$
4) $\tan\theta=\frac{1}{2\sqrt{3}}$

7. A particle of mass m is moving along a trajectory given by
$x = x_0 + a\cos\omega_1 t$
$y = y_0 + b\sin\omega_2 t$
The torque, acing on the particle about the origin, at t = 0 is:
1) $m(-x_0b+y_0a)\omega_1^2\hat{k}$
2) $+my_0a\omega_1^2\hat{k}$
3) $-m(x_0b\omega_2^2-y_0a\omega_1^2)\hat{k}$
4) zero

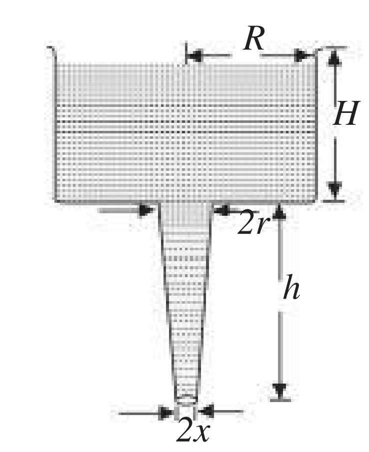
8. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be:
1) $\frac{2GM}{9a^2}$ 2) $\frac{GM}{9a^2}$
3) $\frac{GM}{3a^2}$ 4) $\frac{2GM}{3a^2}$

9. A bottle has an opening of radius a and length b. A cork of length b and radius (a + Δa) where (Δa << a) is compressed to fit into the opening completely (see figure). If the bulk modulus of cork is B and frictional coefficient betwe-en the bottle and cork is p then the force needed to push the cork into the bottle is :



1) (π Bb)a 2) (2π Bb) Δa
3) (π Bb)Δa 4) (4π Bb) Δa

10. Consider a water jar of radius R that has water filled up to height if and is kept on a stand of height h (see figure). Through a hole of radius r (r << R) at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross-section of water stream when it hits the ground is x. Then:



1) $x=r\left(\frac{H}{H+h}\right)^{1/4}$
2) $x=r\left(\frac{H}{H+h}\right)$
3) $x=r\left(\frac{H}{H+h}\right)^2$
4) $x=r\left(\frac{H}{H+h}\right)^{1/2}$

Solutions

1. **2;** $F=\alpha\beta e^{\left(\frac{-x^2}{\alpha KT}\right)}$

$$\left[\frac{x^2}{\alpha KT}\right]=M^0L^0T^0$$

$$\frac{L^2}{[\alpha]ML^2T^{-2}}=M^0L^0T^0$$

$$\Rightarrow[\alpha]=M^{-1}T^2\Rightarrow[F]=[\alpha][\beta]$$

$$MLT^{-2}=M^{-1}T^2[\beta]\Rightarrow[\beta]=M^2LT^{-4}$$

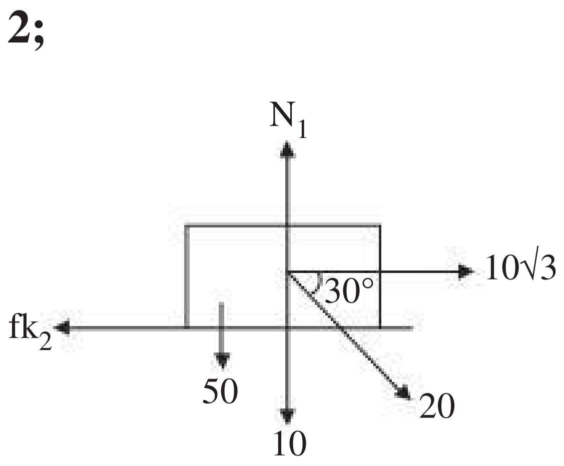
2. **2;**
v = b√x

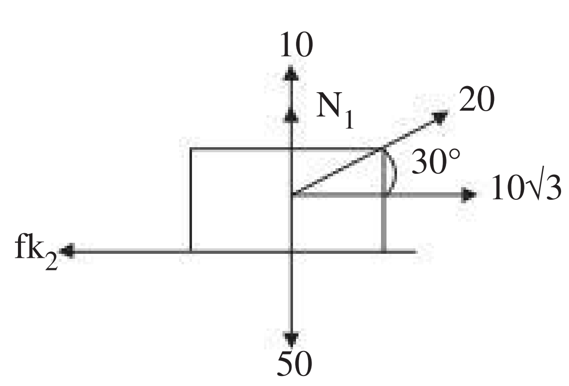
$$\frac{dv}{dt}=\frac{b}{2\sqrt{x}}\frac{dx}{dt};\ a=\frac{bv}{2\sqrt{x}}$$

$$a=\frac{b(b\sqrt{x})}{2\sqrt{x}};$$

$$\frac{dv}{dt}=a=\frac{b^2}{2};v=\frac{b^2}{2}\tau$$

3. **2;**





$N_1 = 60$

$$a_1=\frac{10\sqrt{3}-0.2\quad 60}{5}$$

$N_2 = 40$

$$a_2=\frac{10\sqrt{3}-0.2\quad 40}{5}$$

$a_1 - a_2 = 0.8$

4. **2;**

$$-(g+\gamma v^2)=\frac{dv}{dt}$$

$$-gdt=\frac{g}{\gamma}\left(\frac{dv}{\frac{g}{v}+v^2}\right)$$

Integrating 0 → t and V_0 → 0:

$$-gt=-\sqrt{\frac{g}{\gamma}}\tan^{-1}\left(\frac{V_0}{\sqrt{\frac{g}{\gamma}}}\right)$$

$$t=\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$$

5. **2;**
Angular momentum conservation
$MV_0L = MV_1(L-l)$

$$V_1=V_0\left(\frac{L}{L-l}\right)$$

$W_g + W_P = \Delta KE$

$$-mgl+W_P=\frac{1}{2}m(V_1^2-V_0^2)$$

$$W_p=mgl+\frac{1}{2}mV_0^2\left(\left(\frac{L}{L-l}\right)^2-1\right)$$

$$=mgl+\frac{1}{2}mV_0^2\left(\left(1-\frac{L}{L-l}\right)^{-2}-1\right)$$

now, l << L
By, Binomial approximation

$$=mgl+\frac{1}{2}mV_0^2\left(\left(1+\frac{L}{L-l}\right)^{-2}-1\right)$$

$$=mgl+\frac{1}{2}mV_0^2\left(\frac{2l}{L}\right)$$

$$W_P=mgl+mV_0^2\frac{l}{L}$$

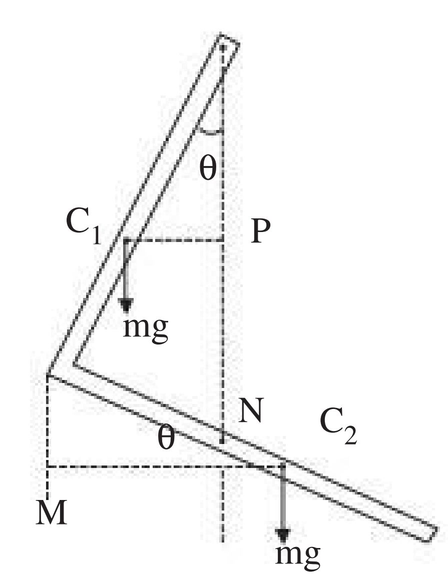
Here, V_0 = Maximum velocity

$$=\omega\quad A=\left(\sqrt{\frac{g}{L}}\right)(\theta_0L)$$

$$So, W_P=mgl+m(\theta_0\sqrt{gL})^2\frac{l}{L}$$

$$=mgl(1+\theta_0^2)$$

6. **2;**



Let mass of one rod is m.
Balancing torque about hinge point.
$mg(C_1P) = mg(C_2N)$

$$mg\left(\frac{L}{2}\sin\theta\right)=mg\left(\frac{L}{2}\cos\theta-L\sin\theta\right)$$

$$\Rightarrow\frac{3}{2}mgL\sin\theta=\frac{mgL}{2}\cos\theta$$

$$\Rightarrow\tan\theta=\frac{1}{3}$$

7. **2;**

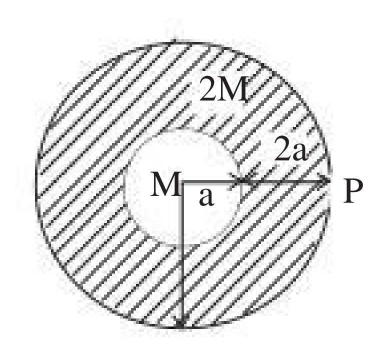
$$\vec{F}=m\vec{a}=m[-a\omega_1^2\cos\omega,t\hat{i}-b\omega_2^2\sin\omega_2t\hat{j}$$

$$\vec{f}_{t=0}=-ma\omega_1^2\hat{i}$$

$$\vec{r}_{t=0}=(x_0+a)\hat{i}+y\hat{j}$$

$$\vec{\tau}=\vec{r}\quad\vec{F}=my_0a\omega_1^2\hat{k}$$

8. **3;**



We use Gauss's Law for gravitation
G.4πr^2 = (Mass enclosed) 4πG

$$G=\frac{3M\,4\pi G}{4\pi(3a)^2}=\frac{GM}{3a^2}$$

9. **4;** $\beta\frac{\Delta V}{V}=\Delta P$
$V_i = \pi(a + \Delta a)^2b$
$V_f = \pi a^2b$
$\Delta V\simeq-2\pi ab\Delta a$

$$\frac{\Delta V}{V}=\frac{-2\pi ab\Delta a}{\pi a^2b}=\frac{-2\Delta a}{a}$$

$$\Rightarrow\Delta P=\frac{2\beta\Delta a}{a}$$

$$\text{Normal Force}=\frac{2\beta\Delta}{a}2\pi a\propto b$$

= 4πβbΔa
friction =   N
= 4π βbΔa

10. **1;** Equation of Continuity :
Mass of the liquid entering per second at A = mass of the liquid leaving per second at B.
$a_1 v_1 = a_2 v_2$
where in
a_1 and a_2 be the area of cross section.

$$A_1v_1=A_2v_2$$

$$\pi r^2\sqrt{2gH}=\pi x^2\sqrt{2g(H+h)}$$

$$\therefore x=r\left(\frac{H}{H+h}\right)^{1/4}$$