

# Find the volume of the sphere?

## MATHS MODEL PAPER (WITH SOLUTIONS) PAPER -2

### SECTION -I

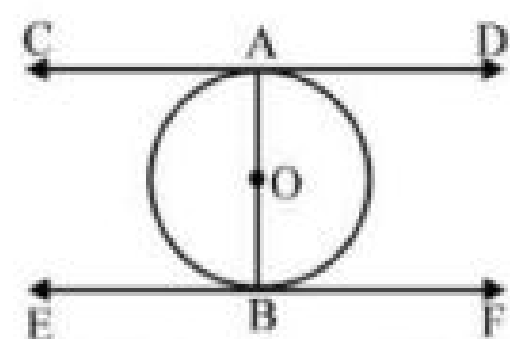
1. Find the value of

$$\cos^2 \theta (1 + \tan^2 \theta) = 1$$

Sol: Given  $\cos^2 \theta (1 + \tan^2 \theta)$

$$= \cos^2 \theta (\sec^2 \theta) = \cos^2 \theta \times \frac{1}{\cos^2 \theta} = 1.$$

2. Prove that the tangents to a circle at the end points of a diameter are parallel.



Sol: OA, is radius, CAD is tangent  $\angle CAO = 90^\circ$  (angle between the radius and tangent) similarly OB is radius, EBF is tangent  $\angle FBO = 90^\circ$  (angle between the radius and tangent) But  $\angle CAO = \angle FBO$  are alternate angles  $\therefore CD \parallel EF$

3. A number  $x$  is chosen at random from the numbers -4, -3, -2, -1, 0, 1, 2, 3, 4. The probability that  $|x| < 3$  is ?

Sol: Total number of outcomes =  $n(S) = 9$  favorable outcomes  $|x| < 3$  are

$$-2, -1, 0, 1, 2, n(E) = 5, P(E) = \frac{n(E)}{n(S)} = \frac{5}{9}$$

4. The total surface of a sphere is 616 sq. cm find the volume of the sphere.

Sol: Total surface of a sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times r^2 = 616$$

$$\Rightarrow r^2 = 616 \times \frac{7}{22} \times \frac{1}{4}$$

$$\Rightarrow r^2 = 7 \times 7 \Rightarrow r = 7 \text{ cm}$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 1437.33 \text{ cub. cm.}$$

5. Write the formula to find median of the grouped data and explain the terms involving in it.

Sol: The median for the grouped data can be found by using the formula

$$\text{Median} = l + \left( \frac{\frac{n}{2} - cf}{f} \right) \times h$$

where,  $l$  = lower limit of the median class.

$n$  = number of observations,

$cf$  = cumulative frequency of class

preceding to the median class,

$f$  = frequency of the median class.

$h$  = class size.

6. Akash said that "In a right triangle two sides are 5cm and 12cm and hypotenuse is 13" do you agree with this statement justify your answer?

Sol: Given larger side = 13 so  $13^2 = 169$ , square of remaining sides  $5^2 = 25$ ,

$$12^2 = 144, \text{ now from the values}$$

$$\text{we know that } 5^2 + 12^2 = 25 + 144 = 169$$

(Pythagoras law states that 'the square of hypotenuse is equal to the sum of square of the other two perpendicular sides),

yes. I agree with Akash statement.

7. Prove that  $\frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = 2\sec^2 \theta$

$$\text{Sol: Given LHS} = \frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta}$$

$$= \frac{1 - \sin \theta + 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)} = \frac{2}{1 - \sin^2 \theta}$$

$$= \frac{2}{\cos^2 \theta} = \text{RHS (}\because \sec^2 \theta = \frac{1}{\cos^2 \theta}\text{)}$$

### SECTION -II

8. Find the ratio of the total surface area to the lateral surface area of a cylinder with base radius 80cm and height 20cm.

Sol: Given radius  $r = 80$  cm and height  $h = 20$  cm

$$\frac{\text{Total surface area of a cylinder}}{\text{Curved surface area of a cylinder}} = \frac{2\pi r(h+r)}{2\pi rh}$$

$$= \frac{h+r}{h} = \frac{20+80}{20} = \frac{100}{20} = \frac{5}{1}$$

9. The angle of elevation of top of a tower from a point 50m away from the base of the tower is  $45^\circ$ . The angle of elevation of top of the flag mounted on the tower  $60^\circ$ . Draw the related diagram.

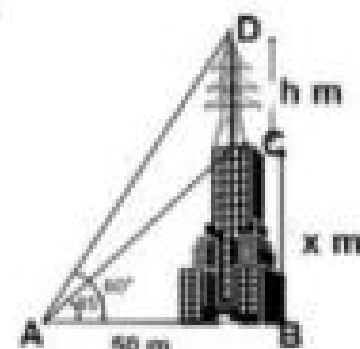
Sol: Let AB = distance between the tower and the observation point.

CD = height of the flag mounted on the tower =  $h$  m.

BC = height of the tower =  $x$  m.

$$\angle CAB = 45^\circ, \angle DAB = 60^\circ$$

## 10th Class Special



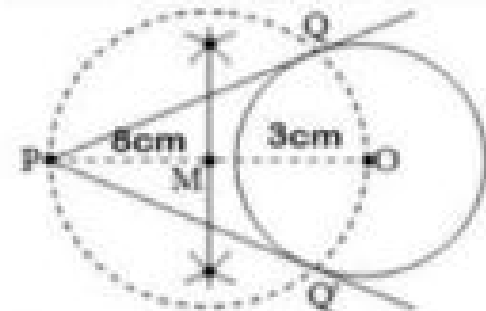
10. A number is selected from the first 50 natural numbers. What if the probability that it is a multiple of 3 or 5?

Sol: Total number of outcomes  $n(S) = 50$  favorable outcomes are 3, 5, 6, 9, 10, 12, 15,

18, 20, 21, 24, 25, 27, 30, 33, 35, 36, 37, 39, 40, 42, 45, 48, 50  $n(E) = 24$

$$\text{probability (multiple of 3 or 5)} = \frac{24}{50} = \frac{12}{25}$$

11. Draw a circle of radius 3cm. From a point 8 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.



Construction steps:

1. Draw a circle of radius 3cm, with centre 'O'
2. Mark point P, 8cm away from the centre.
3. Join OP and draw the perpendicular bisector of OP which intersect at M
4. Draw a circle with centre M, with radius

$MP = MO$ , these circle intersect previous circle at Q and Q'.

5. Join PQ and PQ' we get required tangents.

12. Find the value of 'k' of the following data, if the mean of the distribution is 19.5.

x	5	10	15	20	25	30	35
f	1	2	5	6	3	k	1

Sol:

x	f	fx
5	1	5
10	2	20
15	5	75
20	6	120
25	3	75
30	k	30k
35	1	35
Total	18 + k	330 + 30k

$$\text{Mean } \bar{x} = \frac{\sum fx}{\sum f} = \frac{330 + 30k}{18 + k} = 19.5$$

$$\Rightarrow 330 + 30k = 351 + 19.5k$$

$$\Rightarrow 30k - 19.5k = 351 - 330$$

$$\Rightarrow 10.5k = 21 \Rightarrow k = \frac{21}{10.5} = 2$$

13. If  $\sec 4A = \text{cosec}(A - 20^\circ)$  where  $4A$  is an acute angle, find the value of A.

Sol: Given  $\sec 4A = \text{cosec}(A - 20^\circ)$

$$\sec 4A = \sec(90^\circ - (A - 20^\circ))$$

$$\Rightarrow \sec 4A = \sec(110^\circ - A)$$

$$\Rightarrow 4A = (110^\circ - A)$$

$$\Rightarrow 5A = 110^\circ \Rightarrow A = 22^\circ$$

### SECTION -III

14(a). Two dice are thrown together, write all the outcomes and Find the probability that the product of the numbers on the top of dice is a multiple of (i) a multiple of 6 (ii) multiple of 12 (iii) factor of 36.

Sol: Two dice are thrown together the possible outcomes are  $6 \times 6 = 36$

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

product of the outcomes as multiple of 6 in the following cases (1, 6) (2, 3), (2, 6), (3, 2), (3, 4), (3, 6), (4, 3), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) = 15

$$P(\text{multiple of 6}) = \frac{15}{36} = \frac{5}{12}$$

product of the outcomes as multiple of 12 in the following cases (2, 6), (3, 4), (4, 3), (4, 6), (6, 2), (6, 4), (6, 6) = 7

$$P(\text{multiple of 6}) = \frac{7}{36}$$

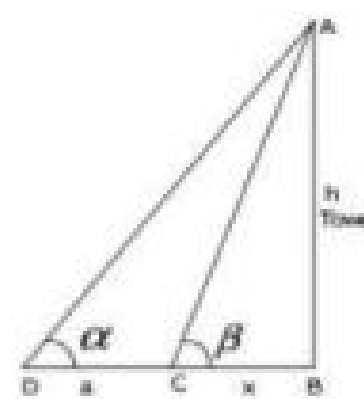
product of the outcome as a factors of 36 in the following cases (1, 1), (1, 2) (1, 3) (1, 4), (1, 6), (2, 1), (2, 2) (2, 3), (2, 6), (3, 1), (3, 2), (3, 3) (3, 4), (3, 6), (4, 1), (4, 3), (6, 1), (6, 2) (6, 3), (6, 6) = 20

$$P(\text{factor of 6}) = \frac{20}{36} = \frac{5}{9}$$

14 (b). The angle of elevation of the top of a tower as observed from a point on the

ground is ' $\alpha$ ' and moving ' $a$ ' meters towards the tower the angle of elevation is ' $\beta$ ' prove that the height of the tower is  $\frac{a \tan \alpha \tan \beta}{\tan \beta - \tan \alpha}$ .

Ans:



Let AB be a tower and height =  $h$  m In  $\Delta$

$$ABC, \tan \beta = \frac{h}{x} \Rightarrow x = \frac{h}{\tan \beta} \dots (1) \text{ In } \Delta ABD$$

$$\tan \alpha = \frac{h}{x+a} \Rightarrow h = (x+a) \tan \alpha \dots (2)$$

substituting x value from (1) in (2)

$$h = \left[ \frac{h}{\tan \beta} + a \right] \tan \alpha$$

$$h = \frac{h \tan \alpha}{\tan \beta} + a \tan \alpha$$

$$h \tan \beta = h \tan \alpha + a \tan \alpha \tan \beta$$

$$h (\tan \beta - \tan \alpha) = a \tan \alpha \tan \beta$$

$$\therefore h = \frac{a \tan \alpha \tan \beta}{\tan \beta - \tan \alpha}$$

15 (a). Construct a  $\Delta ABC$  in which  $AB = 5$  cm and  $\angle B = 60^\circ$  and altitude  $CD = 3$  cm, and then a triangle AQR similar to it whose sides are  $\frac{3}{2}$  of the corresponding sides of  $\Delta ACB$ . (Visualisation and Representation)

Sol:

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For Feedback...

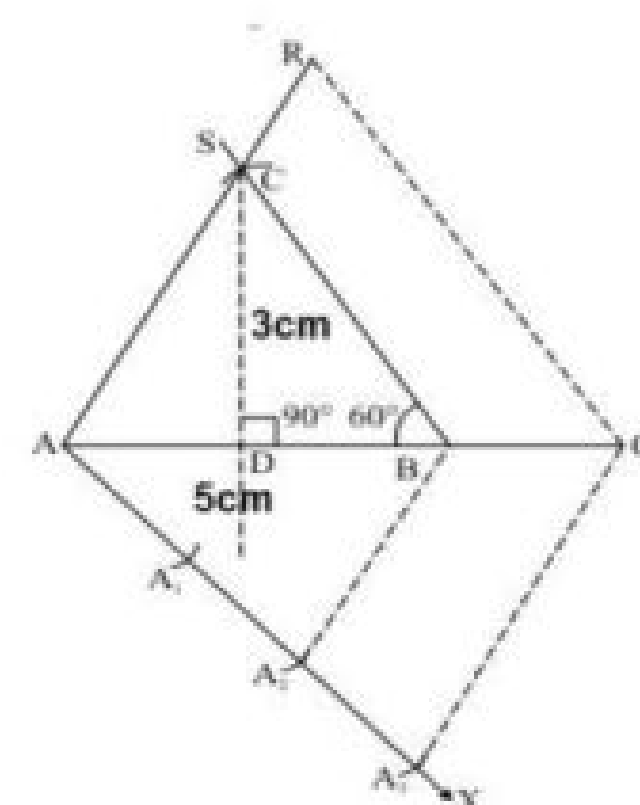
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Construction Steps:

1. Draw a line segment  $AB = 5$  cm. B as

centre draw an angle of  $60^\circ$

2. From A and B construct an altitude of  $CD = 3$  cm, which cut the line BS at point C. join AC we get a triangle ABC.

3. Draw a ray BX make an acute angle with AB.

3. Locate 3 points ( maximum of  $\frac{3}{2}$  )

$A_1, A_2, A_3$ , as  $AA_1 = A_1A_2 = A_2A_3$ .

4. Join  $A_2B$  and draw  $A_3Q \parallel A_2B$  and  $QR \parallel BC$ , we get a required triangle AQR.

5.  $\therefore \Delta ABC: \Delta AQR = 3:2$

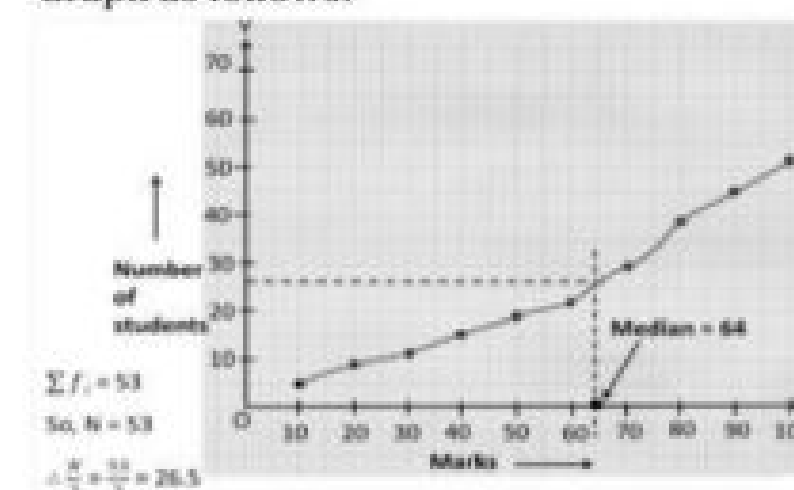
15 (b). The following data indicates the marks of 53 students in mathematics. Draw less than type ogive for the data and hence find median.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
Number of students	5	3	4	3	3	4	7	9	7	8

Sol:

Marks	Number of students	Upper Limits	Less than cumulative frequency
0 - 10	5	10	5
10 - 20	3	20	8
20 - 30	4	30	12
30 - 40	3	40	15
40 - 50	3	50	18
50 - 60	4	60	22
60 - 70	7	70	29
70 - 80	9	80	38
80 - 90	7	90	45
90 - 100	8	100	53

Graph as follows:



from the graph we can notice that median of the data is 64 marks (approximately)

# Find the orthocenter of the triangle?

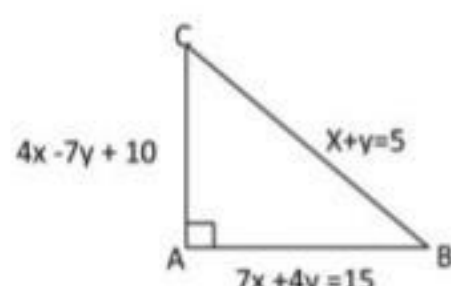
## LOCUS

### 2 Marks

Continued from 15th December

11. Find the orthocenter of the triangle whose sides are given by  $4x-7y+10=0$ ,  $x+y=5$  and  $7x+4y=15$ .

A: Let the three given lines be the sides AB, BC, CA of the triangle respectively. Slope of AB =  $4/7$ ; Slope of CA =  $-7/4$



Product of the above two slopes = -1

Therefore, AB is perpendicular to CA, i.e., The triangle is right-angled at A. In a right-angled triangle, the orthocenter is the vertex containing the right angle, i.e., A. We get the coordinates of A by solving the equation of AB and CA.  $(4x-7y+10=0) \times 4 \Rightarrow 16x-28y+40=0$   $(7x+4y-15=0) \times 7 \Rightarrow 49x+28y-105=0$

On adding the above two equations, we get  $65x-65=0 \Rightarrow x=1$

On substituting this value in the 1<sup>st</sup> equation, we get  $4(1)-7y+10=0 \Rightarrow 7y=14 \Rightarrow y=2$  Therefore, the required answer is (1,2).

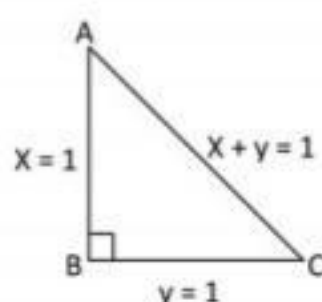
12. Find the circumcenter of the triangle whose sides are  $x=1$ ,  $y=1$  and  $x+y=1$ .

A: Let the three given lines be the sides AB, BC, CA of the triangle respectively.

We have AB is perpendicular to BC, i.e., The triangle is right-angled at B. So, AC is the hypotenuse. In a right-angled triangle, the circumcenter is the midpoint of the hypotenuse, i.e., the midpoint of AC.

AB:  $x=1$ ; BC:  $y=1$ ; CA:  $x+y=1$

On solving, we get A = (1,0) and C=(0,1) The midpoint of AC is  $(\frac{1}{2}, \frac{1}{2})$  which is the required circumcenter.



13. If  $\theta$  is the angle between the lines  $\frac{x}{a} + \frac{y}{b} = 1$  and  $\frac{x}{b} + \frac{y}{a} = 1$ , find the value of  $\sin\theta$  when  $a > b$ .

A:  $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow bx+ay-ab=0$

$\frac{x}{b} + \frac{y}{a} = 1 \Rightarrow ax+by-ab=0$

So, we have  $|\cos\theta| = \frac{|ba+ab|}{\sqrt{b^2+a^2}\sqrt{a^2+b^2}} = \frac{|2ab|}{a^2+b^2}$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - \frac{4a^2b^2}{(a^2+b^2)^2}$

$= \frac{(a^2+b^2)^2 - 4a^2b^2}{(a^2+b^2)^2} = \frac{(a^2-b^2)^2}{(a^2+b^2)^2}$

$\Rightarrow \sin\theta = \frac{a^2-b^2}{a^2+b^2} (\because a > b, \sin\theta > 0)$

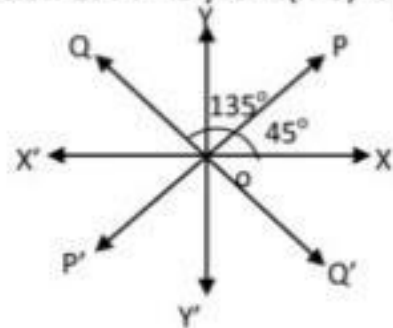
### 4 Marks

1. Find the equation of the straight lines passing through the origin and making equal angles with the co-ordinate axes.

Sol: Case(i):  $PP'$  makes an angle  $45^\circ$  with positive X-axis  $m=\tan 45^\circ = 1$

$PP'$  passes through O(0,0)

Equation of  $PP'$  is  $y-0=1(x-0) \Rightarrow y=x$



Case (ii):  $QQ'$  makes angle  $135^\circ$  with positive X-axis  $m=\tan 135^\circ = \tan(180^\circ - 45^\circ) = -\tan 45^\circ$

Equation of  $QQ'$  is  $y-0=-1(x-0) \Rightarrow y=-x$

2. Find the equation of the straight line passing through (-2,4) and making non-zero intercepts whose sum is zero.

Sol: Equation of the line in the intercept form is  $\frac{x}{a} + \frac{y}{b} = 1$ . Given of the line  $a+b=0 \Rightarrow b=-a$

Equation of the line  $\frac{x}{a} - \frac{y}{a} = 1$

$\Rightarrow x-y=a$  The line passes through P(-2,4)  $\therefore$

$-2-4=a \Rightarrow a=-6$

Equation of the required line  $x-y=-6 \Rightarrow x-y+6=0$

3. Line L has intercepts a and b on the axes of co-ordinates. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q on the transformed axes. Prove that  $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$ .

Sol: Equation of the line in the old system in the intercept form is  $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$

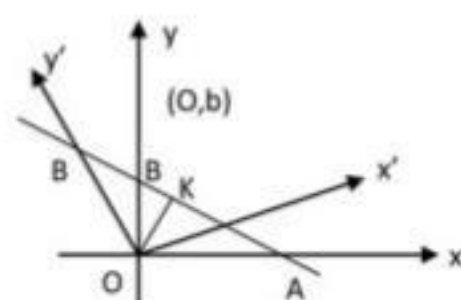
Length of the perpendicular from origin =  $\frac{|0+0-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$

..... (1)

Equation of the line in the second system in the intercept form is  $\frac{x'}{p} + \frac{y'}{q} = 1 \Rightarrow \frac{x'}{p} + \frac{y'}{q} - 1 = 0$

Length of the perpendicular from origin

$= \frac{|0+0-1|}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}} \dots\dots\dots (2)$



Since the origin and the given line remain unchanged we have from (1) and (2)

$\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

## INTERMEDIATE SPECIAL JUNIOR

4. If  $3a+2b+4c=0$ , then show that the equation  $ax+by+c=0$  represents a family of concurrent straight lines and find the point of concurrency.

Sol: Given condition of a,b the line  $ax+by+c=0$

$(\frac{3}{4})a + (\frac{1}{2})b + c = 0$

For all values of a,b the line  $ax+by+c=0$  passes through the point  $(\frac{3}{4}, \frac{1}{2})$  equation  $ax+by+c=0$  represents a family of concurrent lines point of concurrency is  $(\frac{3}{4}, \frac{1}{2})$

5. The line  $\frac{x}{a} - \frac{y}{b} = 1$  meets the X-axis at P. Find the equation of the line perpendicular to the line at P.

Sol: Equation of PQ is  $\frac{x}{a} - \frac{y}{b} = 1$

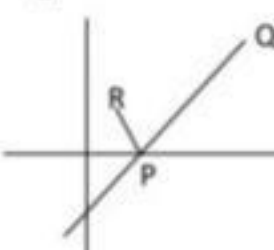
Equation of X-axis is  $y=0 \Rightarrow \frac{x}{a} = 1 \Rightarrow x=a$

Co-ordinates of P are (a,0) PR is perpendicular to PQ

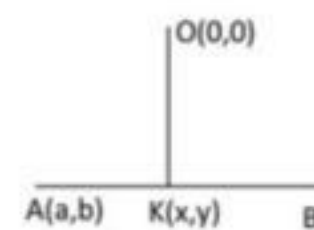
Equation of PR =  $\frac{x}{b} + \frac{y}{a} = k$  This line PR passes through P is

$(\frac{a}{b}, 0) + 0 = k \Rightarrow k = \frac{a}{b}$

Equation of PR is  $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$



6. Find the locus of the foot of the perpendicular from the origin to a variable straight line which always passes through a fixed point (a,b).



Sol: Suppose m is the slope of the line AB

Equation of AB is  $y-b=m(x-a) \Rightarrow mx-my+by-b+ma=0$  ..... (1)

OK is perpendicular to AB and passes through the origin O. Suppose co-ordinates of K are (x,y) Equation of OK is  $x+my=0$  ..... (2)

$\frac{x}{-m(b-ma)} = \frac{y}{b-ma} = \frac{1}{m^2+1}$

$x = \frac{-m(b-ma)}{1+m^2}, y = \frac{b-ma}{1+m^2}$

$\frac{y}{x} = \frac{b-ma}{-m(b-ma)} = -\frac{1}{m} \Rightarrow m = -x/y$

Substituting in (1)

$-\frac{x^2}{y} - y + b + \frac{x}{y} \cdot a = 0$

$-x^2 - y^2 + by + ax = 0$

Or  $x^2 + y^2 - by - ax = 0$

Locus of K is  $x^2 + y^2 - ax - by = 0$

7. Find the equation of the straight lines passing through the point (-10,4) and making an angle  $\theta$  with the line  $x-2y=10$  such that  $\tan\theta=2$ .

Sol: Equation of QR is  $x-2y=10$

Suppose slope of PQ is m, PQ passes through P(-10,4)

Equation of PQ is  $y-4=m(x+10) \Rightarrow y-4=mx+10m$

i.e.,  $mx-y+(10m+4)=0$

..... (1)

$\tan\theta=2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{|a_1a_2+b_1b_2|}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}} \Rightarrow \frac{1}{\sqrt{5}} = \frac{|m+2|}{\sqrt{1+4m^2+1}}$

Squaring and cross multiplying

$m^2+1=(m+2)^2 \Rightarrow m^2+1=m^2+4m+4$

$4m+3=0 \Rightarrow m=-\frac{3}{4}$

Case (i) : Co-efficient of  $m^2=0$

$\Rightarrow$  One of the roots is  $\infty$

Hence PR is a vertical line

$\therefore$  Equation of PR is  $x+10=0$

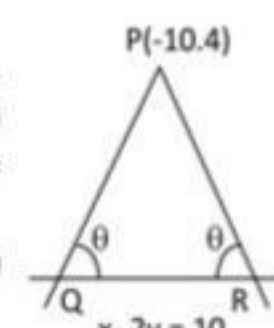
Case(ii) :  $m = -\frac{3}{4}$

Substituting in (i)

Equation of PQ is

$-\frac{3}{4}x - y + (-\frac{30}{4} + 4) = 0$

$\frac{-3x-4y-14}{4} = 0 \Rightarrow 3x+4y+14=0$



8. Let  $\overline{PS}$  be the median of the triangle with vertices P(2,2), Q(6,-1) and R(7,3). Find the equation of the straight line passing through (1,-1) and parallel to the median  $\overline{PS}$ .

Sol: P(2,2), Q(6,-1) R(7,3) are the vertices of  $\triangle ABC$ ; S is the mid point of QR

Co-ordinates of S are

$(\frac{6+7}{2}, \frac{-1+3}{2}) = (\frac{13}{2}, 1)$

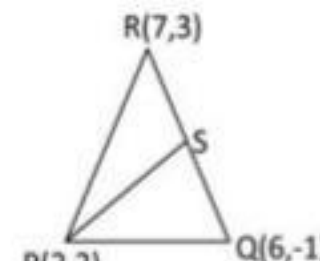
Slope of PS =  $\frac{1-2}{\frac{13}{2}-2} = -\frac{2}{9}$

AB is parallel to PS and passes through A(1,-1)

Equation of AB is  $y+1=-\frac{2}{9}(x-1)$

$9y+9=-2x+2$

$2x+9y+7=0$



9. Find the points on the line  $3x-4y-1=0$  which are at a distance of 5 units from the point (3,2)

Sol: Equation of the line in the symmetric form is

$\frac{x-3}{\cos\alpha} = \frac{y-2}{\sin\alpha} = r$

Co-ordinates of the point P are  $(3+r\cos\alpha, 2+r\sin\alpha)$

$P$  is a point on  $3x-4y-1=0$

$3(3+5\cos\alpha) - 4(2+5\sin\alpha) - 1 = 0$

$9+15\cos\alpha - 20\sin\alpha - 1 = 0$

$\tan\alpha = \frac{3}{4}$

Case (i)  $\cos\alpha = \frac{4}{5}, \sin\alpha = \frac{3}{5}$

Case (ii)  $\cos\alpha = -\frac{4}{5}, \sin\alpha = -\frac{3}{5}$

Case (i) : co-ordinates of P are

$(3+5\frac{4}{5}, 2+5\frac{3}{5}) = (7,5)$

Case(ii): co-ordination of P are

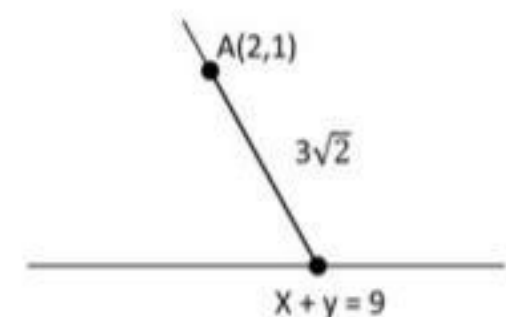
$(3-5\frac{4}{5}, 2-5\frac{3}{5}) = (-1,-1)$

10. A straight line L is drawn through the point A(2,1) such that its point of intersection with the straight line  $x+y=9$  is at a distance  $3\sqrt{2}$  from A. Find the angle which the line L makes with the positive direction of the X-axis.

Sol: Suppose  $\alpha$  is the angle made by the line through A, with the positive X-axis.

Any point on the line is

$(x_1 + r\cos\alpha, y_1 + r\sin\alpha)$



$= (2+3\sqrt{2}\cos\alpha, 1+3\sqrt{2}\sin\alpha)$

This is a point on the line  $x+y=9$

$2+3\sqrt{2}\cos\alpha + 1+3\sqrt{2}\sin\alpha = 9$

$3\sqrt{2}(\cos\alpha + \sin\alpha) = 6$

## విజేత

For Feedback...

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$\cos\alpha + \sin\alpha = \frac{6}{3\sqrt{2}} = \sqrt{2}$

$\frac{1}{\sqrt{2}} \cdot \cos\alpha + \frac{1}{\sqrt{2}} \sin\alpha = 1$

$\cos\alpha \cdot \cos 45^\circ + \sin\alpha \cdot \sin 45^\circ = 1$

$\cos(\alpha - 45^\circ) = \cos 0^\circ$

$\alpha - 45^\circ = 0 \Rightarrow \alpha = 45^\circ = \frac{\pi}{4}$

### 7 Marks

1. Find the equation of the straight line passing through the point (3,4) and making X and Y intercepts which are in the ratio 2:3

Sol: Equation of the line in the intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$

Given  $\frac{a}{b} = \frac{2}{3} \Rightarrow b = \frac{3a}{2}$

Equation of the line is  $\frac{x}{a} + \frac{2y}{3a} = 1$

This line passes through P(3,4)

$9-8=3a \Rightarrow 3a=1$

Equation of the required line is  $3x+2y=1$

$\Rightarrow 3x+2y-1=0$

2. A straight line through P(3,4) makes an angle of  $60^\circ$  with the positive direction of the X-axis. Find the co-ordinates of the points on that line which are 5 units away from P.

Sol: Co-ordinates of any point on the line Q are

$(x_1 + r\cos\theta, y_1 + r\sin\theta)$

Given  $(x_1, y_1) = (3,4)$  i.e.,  $x_1 = 3, y_1 = 4$

$\theta = 60^\circ \Rightarrow \cos\theta = \cos 60^\circ$

$= \frac{1}{2}, \sin\theta = \sin 60^\circ = \frac{\sqrt{3}}{2}$

Case(i):  $r = 5$

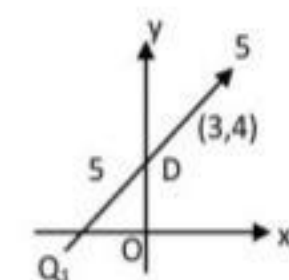
Co-ordinates of Q are  $(3+5\frac{1}{2}, 4+5\frac{\sqrt{3}}{2})$

$= (\frac{11}{2}, \frac{8+5\sqrt{3}}{2})$

Case (ii):  $r = -5$

Co-ordinates of Q are  $(3+5\frac{1}{2}, 4+5\frac{\sqrt{3}}{2})$

$= (\frac{1}{2}, \frac{8-5\sqrt{3}}{2})$



3. A straight line through Q( $\sqrt{3}, 2$ ) makes an angle  $\frac{\pi}{6}$  with the positive direction of the X-axis. If the straight line intersects the line  $\sqrt{3}x-4y+8=0$  at P, find the distance of PQ. Note: AB and PQ are not perpendicular. So we have to follow the first method only.

Sol: PQ makes an angle  $\frac{\pi}{6}$  with the positive direction of X-axis.

$m = \text{slope of PQ} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

PQ passes through Q( $\sqrt{3}, 2$ )

Equation of PQ is  $y-2 = \frac{1}{\sqrt{3}}(x-\sqrt{3})$

$\sqrt{3}y-2\sqrt{3}=x-\sqrt{3}$

$x-\sqrt{3}y=-\sqrt{3}$  ..... (1)

Equation of AB is  $\sqrt{3}x-4y+8=0$

$\sqrt{3}x-4y=-8$

$(1) \times \sqrt{3} \Rightarrow \sqrt{3}x-3y=-3$

Subtracting  $-y=-5$

From (1),  $x = \sqrt{3}y - \sqrt{3} = 5\sqrt{3} - \sqrt{3}$

$= 4\sqrt{3}$  co-ordinates of P are  $(4\sqrt{3}, 5)$

Q are  $(\sqrt{3}, 2)$

$PQ^2 = (4\sqrt{3}-\sqrt{3})^2 + (5-2)^2$

$= 27+9=36$   $PQ=6$  units.