Find two consecutive positive integers...

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IMPORTANT QUESTIONS

1. Find the HCF of 2400 and 2015. By using Euclid algorithm.

A. By Euclid Algorithm

$2400 = 2015 \times 1 + 385$
$2015 = 385 \times 5 + 90$
$385 = 90 \times 4 + 25$
$90 = 25 \times 3 + 15$
$25 = 15 \times 1 + 10$
$15 = 10 \times 1 + 5$
$10 = 5 \times 2 + 0$
$\therefore$ HCF = 5

2. List all the sub-sets of the set S. Where S = {x/x is a letter of the word "monsoon"}

A. The Roster form of the given set in
S = {m, o, n, s}
Sub sets of S are
∅, S, {m}, {o}, {n} ,{s},
{m, o}, {m, n} {m, s},
{o, n} {o, s}, {n, s},
{m, o, n}, {m, o, s},
{m, n, s}, {o, n, s}

3. Check whether $-\sqrt{12}, \sqrt{12}$ are the zeroes of the Polynomial $2x^2 - 24$.

A. Let $p(x) = 2x^2 - 24$
for $p(-\sqrt{12})^2 = 2(-\sqrt{12})^2 - 24$
$= 24 - 24 = 0$
and $p(\sqrt{12}) = 2(\sqrt{12})^2 - 24$
$= 24 - 24 = 0$
As $p(-\sqrt{12}) = 0$ and $p(\sqrt{12}) = 0 \Rightarrow -\sqrt{12}$ and $\sqrt{12}$ are the zeroes of $p(x)$

4. Srilatha says that the order of the polynomial $(x^3 - 6)(x^2 + 4)$ is 6. Do you agree with her? How?

A. $(x^3 - 6)(x^2 + 4) = x^3(x^2 + 4) - 6(x^2 + 4)$
$= x^3.x^2 + 4x^3 - 6x^2 - 24$
$= x^5 + 4x^3 - 6x^2 - 24$ [$\because x^m. x^n = x^{m+n}$]
As the power of the given polynomial is 5. I do not agree with Srilatha.

5. Nagaraju stated that (6, 4), and (–6, 4) are the points on the line which is parallel to X-axis. Discuss.

A. Let M(6, 4) and N(–6, 4) are the two given points. slope of $\overline{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-4}{-6-6} = 0$
As the slope of the line is zero. It is parallel to x-axis and also the points of the form (x, k), (y, k).
$\therefore$ We can conclude that the statement given by Nagaraju is true.

6. Find the roots of the Quadratic equation $x^2 + 4x + 5 = 0$ if they exist.

A. For the given quadratic equation, $x^2 + 4x + 5 = 0$
a = 1, b = 4, c = 5.
Discriminant = $b^2 - 4ac$
$= (4)^2 - 4(1)(5)$
$= 16 - 20 = -4 < 0$
as, $b^2 - 4ac < 0$
$\Rightarrow \sqrt{b^2 - 4ac}$ has no real roots.
$\therefore$ There are no real roots for the given equation.

7. How can we say $7 \times 13 \times 23 + 7$ and $5 \times 7 \times 9 \times 11 + 11$ are composite numbers.

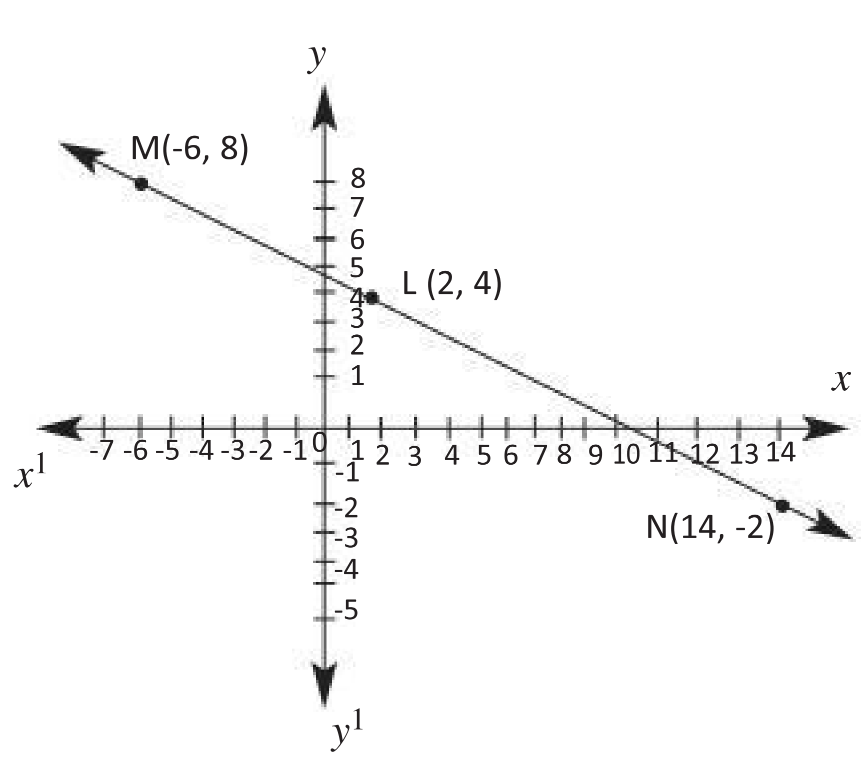
A. Given numbers are $7 \times 13 \times 23 + 7 = 7(13 \times 23 + 1) = 7 \times 300$
and similarly $5 \times 7 \times 9 \times 11 + 11 = 11(5 \times 7 \times 9 + 1) = 11 \times 316$
i.e. the two given numbers can be written as the product of two numbers
$\Rightarrow$ They are composite.

8. Find two consecutive positive integers, sum of whose squares is 1201.

A. Let x and $(x + 1)$ be the two positive consecutive integers.
Where $x^2 + (x + 1)^2 = 1201$
$\Rightarrow x^2 + x^2 + 2x + 1 = 1201$
$\Rightarrow 2x^2 + 2x + 1 = 1201$
$\Rightarrow 2x^2 + 2x - 1200 = 0$
$\Rightarrow x^2 + x - 600 = 0$
$\Rightarrow x^2 + 25x - 24x - 600 = 0$
$\Rightarrow x(x + 25) - 24(x + 25) = 0$
$\Rightarrow (x + 25)(x - 24) = 0$
$\Rightarrow x + 25 = 0$ or $x - 24 = 0$
$\Rightarrow x \neq -25$ or $x = 24$
as $x = 24$, $x + 1 = 24 + 1 = 25$
$\therefore$ 24 and 25 are the two required consecutive positive integers.

9. Plot L(2, 4) M(–6, 8) N(14, –2) on the coordinate plane and join them. what do you observe?

A. The three given points are
L(2, 4), M(–6, 8)
and N(14, -2)
Scale:
On the x-axis 1cm = 1 unit
On the y-axis 1cm = 1 unit



From the graph, I observed that the three given points are collinear.

10. Show that $3 + 2\sqrt{6}$ is an irrational number

A. We need to prove $3 + 2\sqrt{6}$ is an Irrational number. Let us assume that $3 + 2\sqrt{6}$ is a rational number
$\Rightarrow$ There exist two integers p and q such that $3 + 2\sqrt{6} = p/q$
$\Rightarrow 2\sqrt{6} = p/q - 3$
$\sqrt{6} = ½ [p/q - 3]$
$\sqrt{6} = \frac{p - 3q}{2q}$ which is a rational number
this is a contradiction to our assumption that $3+2\sqrt{6}$ is a rational number. As $\sqrt{6}$ is a irrational number.
Hence $3 + 2\sqrt{6}$ is an irrational number
Hence proved.

11. In an Arithmetic progression the last term is 28 and the sum of 9 terms is 144 then find the first term.

A. Given that in an arithmetic progression
$l = 28$
$S_9 = 144$, a = ?
we have, $S_n = \frac{n}{2}[a + l]$
$\Rightarrow 144 = \frac{9}{2}[a + 28]$
$\Rightarrow a + 28 = 144\ \frac{2}{9}$
$a + 28 = 32 \Rightarrow a = 4$

12. If (7, –2) (3, k) and (5, 1) are collinear, then find the value of k.

A. Given that L(7, –2), M(3, k) and N (5, 1) are collinear.
$\Rightarrow$ area of ΔLMN = 0
$\Rightarrow ½ |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$
$\Rightarrow ½|7(k - 5) + 3[1-(-2)] + 5(-2-k)| = 0$
$\Rightarrow |7k - 35 + 9 - 10 - 5k| = 0$
$\Rightarrow |2k - 36| = 0$
$\Rightarrow 2k = 36 \Rightarrow k = 18$

13. Can we draw a triangle with the vertices (13, 14), (5, 8) and (1, 5)? Specify the reason why?

A. Let L(13, 14), M(5, 8) and N(1, 5) are the given points.
Now,
Distance between the two points
L and M. i.e. $LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5-13)^2 + (8-14)^2}$
$= \sqrt{64 + 36} = \sqrt{100} = 10$
Similarly,
$MN = \sqrt{(1-5)^2 + (5-8)^2}$
$= \sqrt{16 + 9} = \sqrt{25} = 5$
and
$LN = \sqrt{(1-13)^2 + (5-14)^2}$
$= \sqrt{144 + 81} = \sqrt{225} = 15$
We have LM + MN = LN
In a triangle, the sum of any two sides must be greater than the third side.
Hence the given 3 points cannot form a triangle.

14. Find the zeros of the polynomial $P(m) = 4m^2 - 4m + 1$ and also verify the relationship between the zeroes and the coefficients.

A. Let $p(m) = 4m^2 - 4m + 1$ be the given polynomial.
We have $4m^2 - 4m + 1 = 4m^2 - 2m - 2m + 1$
$= 2m[2m-1] - 1[2m-1]$
$= (2m - 1)(2m - 1)$
$= (2m - 1)^2$
For $4m^2 - 4m + 1 = 0$,
where a = 4, b = –4, c = 1
$\Rightarrow (2m - 1)^2 = 0$
$\Rightarrow m = ½$
$\therefore$ ½ and ½ are the two zeroes of p(m)
Now, sum of the zeroes = ½ + ½ = 1
$\frac{-b}{a} = \frac{-(-4)}{4} = 1$
$\therefore$ sum of the zeroes
$= \frac{-b}{a} = -\left[\frac{\text{Co-efficient of m}}{\text{Co-efficient of m}^2}\right]$
Product of the zeroes = ½×½ = ¼
c/a = ¼
$\therefore$ Product of the zeroes
$= \frac{c}{a} = \frac{\text{constant}}{\text{Co-efficient of } x^2}$
Hence verified.

15. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x + 2$ if $\sqrt{2}$ and $-\sqrt{2}$ are the two zeroes of the polynomial.

A. Let $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x + 2$
as $\sqrt{2}$ are the zeroes of $f(x)$
$\Rightarrow (x - \sqrt{2}), (x + \sqrt{2})$ are the factors of $f(x)$
and $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ also a factor of $f(x)$

$x^2-2)\ 2x^4 - 3x^3 - 3x^2 + 6x + 2\ (2x^2 - 3x - 1$
$-2x^4 \quad -2x^2$
$+$
$-3x^3 - x^2 + 6x + 2$
$+3x^3 \quad \pm 6x$
$-x^2 + 2$
$-x^2 + 2$
$+ \quad -$
0

$\Rightarrow f(x) = (x^2 - 2)(2x^2 - 3x - 1)$
$2x^2 - 3x + 1 = 2x^2 - 2x - x + 1$
$= 2x[x-1] - 1[x-1]$
$= (2x-1)(x-1)$
if $f(x) = 0$
$\Rightarrow 2x - 1 = 0$ or $x - 1 = 0$
$\Rightarrow x = ½$ or $x = 1$
$\therefore -\sqrt{2}, \sqrt{2}$, 1 and 1/2 are the factors of $f(x)$