Do you agree with her statement?

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MODEL PAPER-I
With Solutions

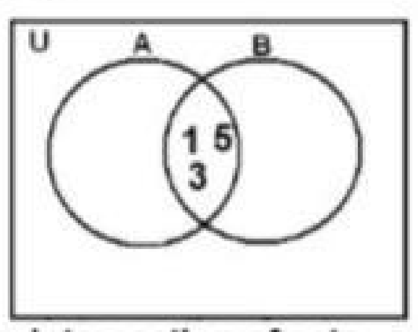
SECTION-I

1. Anusha says that 2310 is a product of consecutive prime numbers. Do you agree with her statement explain?

Sol: Yes. 2310 can be expressed as a product of consecutive prime numbers as $2310 = 2 \times 3 \times 5 \times 7 \times 11$.

2. If A = {1, 2, 3, 4, 5} and B = {1, 3, 5} then find $A \cap B$ using Venn diagram?

Sol:



intersection of sets

3. If α, β are the zeros of the polynomial x^2-2x-8 then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

Sol: Given polynomial $P(x) = x^2-2x-8$

where a = 1, b = −2, c = −8,

α, β are the zeros of the polynomial

then $\alpha + \beta = \frac{-b}{a} = \frac{-(-2)}{1} = 2$

$\alpha\beta = \frac{c}{a} = \frac{-8}{1} = -8$

now $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{2}{-8} = \frac{1}{-4}$

4. For what value of 'k' the pair of equations 3x + 4y + 2 = 0, 9x +12y + k =0 represent coincident lines.

Sol: The given equations 3x + 4y + 2 = 0, 9x +12y + k = 0, where

$a_1 = 3$ b_1= 4, c_1 = 2, a_2 = 9, b_2 = 12, c_2 = k

We know that if the pair of lines coincident

then $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2} \Rightarrow \frac{3}{9} = \frac{4}{12} = \frac{2}{k}$

=> 4k = 24 => k = 6

5. Find the nature of the roots of the equation $3x^2 - 4\sqrt{3}x + 4 = 0$

Sol: Given equation $3x^2 - 4\sqrt{3}x + 4 = 0$ of the form $ax^2 + bx + c = 0$

where, a = 3 and b = $-4\sqrt{3}$, c = 4 then

Discriminant = $b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$ $= 16 \times 3 - 12 \times 4 = 48 - 48 = 0$ so the roots are real and equal.

6. For what value of k, k + 2, k + 6 are in GP?

Sol. If the given terms are in GP then

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{k+2}{k} = \frac{k+6}{k+2}$

=> $(k+2)^2$= k (k +6)

=>$k^2 + 4k + 4 = k^2 + 6k$

=> 4k −6k = −4 => −2k = −4 => k = 2

7. Find the centriod of the triangle whose vertices are (2, −3)(4, 6) and (−2, 8).

Sol: We know that the centriod of the triangle is

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

given points are (2, −3)(4, 6) and (−2, 8)

the centriod is $\left(\frac{2+4-2}{3}, \frac{-3+6+8}{3}\right) = \left(\frac{4}{3}, \frac{11}{3}\right)$

SECTION - II

8. If the points A (6, 1), B (8, 2), C (9, 4) and D (p, 3) are the vertices of a parallelogram, taken in order. Find the value of p?

Sol: we know that the diagonals of a parallelogram bisect each other.

So Midpoint of AC = Midpoint of BD

Sol: we know that the diagonals of a parallelogram bisect each other.

So Midpoint of AC = Midpoint of BD

$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$

$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$

$\Rightarrow \frac{15}{2} = \frac{8+p}{2} \Rightarrow 15 = 8 + p$

=> p = 15 – 8 = 7 => p = 7

9. If A = { x/x is a natural number}, B = {x/x even number of natural number set} find AUB and A−B?

Sol: A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10,.....}

B= {2, 4, 6, 8, 10,....}

AUB ={ 1, 2, 3, 4, 5, 6, ...} = A ,

A−B = { 1, 3, 5, 7, 9,.........}

10th Class
Special

10. If you simplify $\frac{2\sqrt{180}+3\sqrt{80}}{\sqrt{5}}$ will be rational or irrational? Justify your answer?

Sol: $\frac{2\sqrt{180}+3\sqrt{80}}{\sqrt{5}}$

$= \frac{2\sqrt{3\times3\times2\times2\times5}+3\sqrt{2\times2\times2\times2\times5}}{\sqrt{5}}$

$= \frac{2\times3\times2\sqrt{5}+3\times2\times2\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}(12+12)}{\sqrt{5}}$ = 24 is a rational number

$\therefore \frac{2\sqrt{180}+3\sqrt{80}}{\sqrt{5}}$

is a rational number on simplification.

11. Find the number of terms between 100 and 1000 which are divisible by 9.

Sol: The numbers between100 and1000 which are divisible by 9 are

108, 117,.....................999

we know that a_n = a+(n−1)d

=> 999 = 108+(n−1)9

=> 999 −108 = (n−1)9

=> 891 = (n−1)9

=> $\frac{891}{9}$ = n−1 => 99 = n−1 => n = 100

12. If α and β are the zeros of the quadratic polynomial $p(x) = x^2-px + q$ find the values of (i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha}+\frac{1}{\beta}$

Sol: Since α and β are the zeros of the quadratic polynomial $p(x) = x^2- px + q$

then $\alpha + \beta = \frac{-b}{a} = \frac{-(-p)}{1}$ = p

$\alpha\beta = \frac{c}{a} = \frac{q}{1}$ = q

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = p^2 - 2q$

(ii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{p}{q}$

13. The sum of the squares of two consecutive natural numbers is 421. Find the numbers.

Sol: Let the two consecutive natural numbers be x, x+1 then according to the sum $x^2 + (x+1)^2 = 420$

=> $x^2 + x^2 + 2x + 1 - 420 = 0$

=> $2x^2 + 2x - 420 = 0 \Rightarrow x^2 + x - 210 = 0$

=> $x^2 + 15x - 14x - 210 = 0$

=> x (x + 15) – 14 (x -15) = 0

=> (x−14)(x +15) =0 => x = 14 or x = −15 (−15 is rejecting because x is a natural number)

consider x = 14 then x +1 = 14 +1 = 15

the required numbers are 14, 15

SECTION - III

14(a) Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1,9m + 8.

Sol: Let a be any positive integer and b = 3, $a = 3q+r$, where q>0 and $0<r<3$

$\therefore a = 3q$ or $3q$+1 or $3q$+2

We have the three cases.

Case1: when $a= 3q \Rightarrow a^3 = (3q)^3 = 27q^3$

=> 9 $(3q^3)$ = 9m where m =$3q^3$

Case 2: when a = 3q+1 => $a^3 = (3q+1)^3$

=> $a^3 = 27q^3 + 27q^2 + 9q + 1$

= $9(3q^3 + 3q^2 + q)$+1

= 9m+1, where $m = (3q^3 + 3q^2 + q)$

Case 3: When a = 3q+2

=> $a^3 = (3q+2)^3$

= $27q^3 + 54q^2 + 36q + 8$

= $9(3q^3 + 6q^2 + 4q)$+8

= 9m+8, where m = $(3q^3 + 6q^2 + 4q)$

∴ The cube of any positive integer is of the form 9m, 9m+1,9m+8.

14(b) A = {x/x is a even number, x≤10} B = {x/x = 2y + 1, y∈ W and y≤ 9} then Find (i) AUB (ii) A ∩ B (iii) A−B (iv) B−A

Sol: A = {2, 4, 6, 8, 10},

B= {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} now

AUB ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19} A ∩ B = { }, A−B={ 2, 4, 6, 8, 1} = A

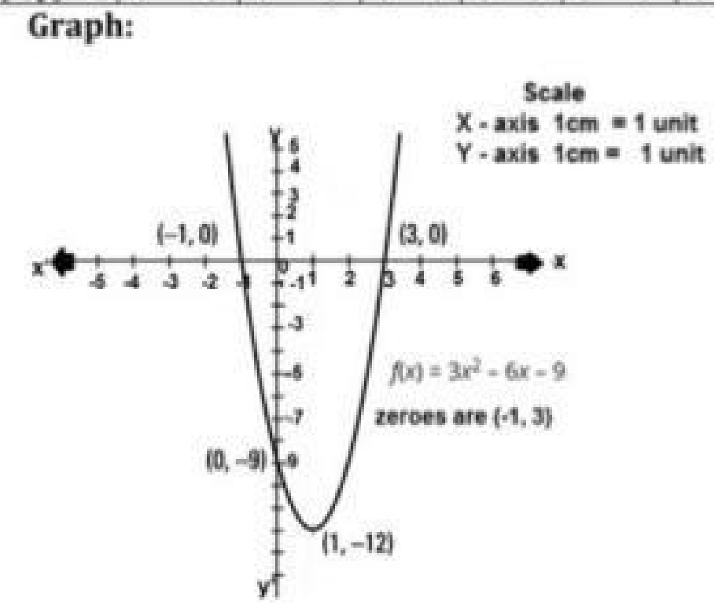
B -A = { 1, 3, 5, 7, 9, 11, 13, 15, 17, 19} = B

15 (a) Draw the graph of the polynomial $p(x) = 3x^2 - 6x - 9$ and find the zeroes from the graph.

Sol: Given polynomial is $3x^2 - 6x - 9$

x	-3	-2	-1	0	2	3
$3x^2$	27	12	3	0	12	27
-6x	18	12	6	0	-12	-18
-9	-9	-9	-9	-9	-9	-9
y = $x^2 - 2x - 3$	36	15	0	-9	-9	0
(x, y)	(-3,36)	(-2,15)	(-1,0)	(0, 9)	(2, -9)	(3, 0)

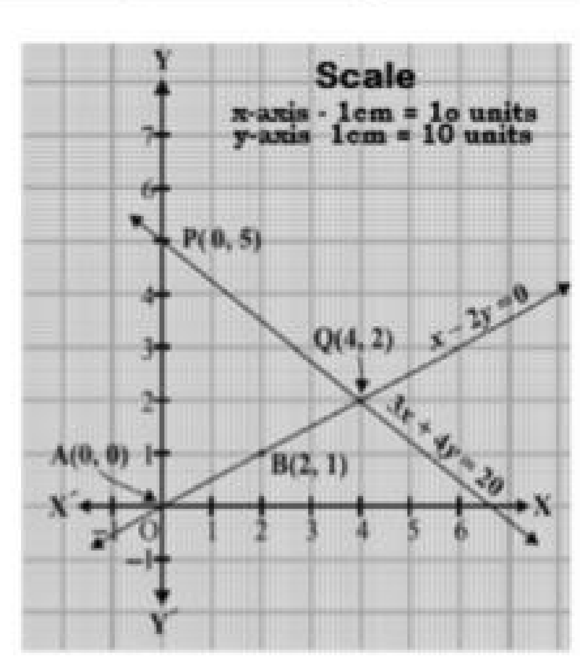
Graph:



15(b) Solve the following system of equations x - 2y = 0, 3x + 4y = 20

x	x-2y =0=> y = $\frac{x}{2}$	(x, y)
0	Y =0/2 = 0	(0, 0)
2	Y =2/2 = 1	(2, 1)
4	Y =4/2 = 2	(4, 2)

x	3x+4y = 20 => y = $\frac{20-3x}{4}$	(x, y)
0	y = $\frac{20-0}{4}$ = 5	(0, 5)
4	Y = $\frac{20-3(4)}{4}$ = 2	(4, 2)
6	Y = $\frac{20-3(6)}{4}$ =	(6, 0.5)



Solution set of the given equations is (4, 2)

16(a). The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$ Find the fraction.

Sol: Let the numerator of the fraction be x then denominator is 2x +1 and

the fraction is $\frac{x}{2x+1}$,

reciprocal of the fraction is $\frac{2x+1}{x}$

according to the sum $\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}$

=> $\frac{x2+(2x+1)^2}{x(2x+1)} = \frac{58}{21}$

=> $\frac{x2+4x^2+4x+1}{2x^2+x} = \frac{58}{21}$

=>$21(5x^2 + 4x + 1) = 58(2x^2 + x)$

=>$105x^2 + 84x + 21 = 116x^2 + 58x$

=>$116x^2 + 58x - 105x^2 - 84x - 21 = 0$

=>$11x^2 - 26x - 21 = 0$

=>$11x^2 - 33x + 7x - 21 = 0$

=>11x (x − 3) + 7(x − 3) = 0

=>(x − 3)(11x + 7) = 0

=> x −3 =or 11x + 7 = 0 => x = 3 or x = $\frac{-7}{11}$

if x =3 then 2x+1 = 2(3) +1 = 7

∴ the required fraction is $\frac{3}{7}$

16(b). The numbers whose sum is 15 are in A.P. if 8, 6 and 4 are added to them respectively then these are in G.P. Find the numbers.

Sol: Let the three numbers are in the AP re a−d and a + d, sum =a – d + a + a + d = 15

=> 3a = 15 => a = 5

according to the sum

a – d + 8, a + 6, a + d + 4 are in GP

=> 5 – d + 8, 5 + 6, 5 + d + 4 are in GP

=> 13 – d ,11, 9 + d are in GP

$(11)^2 = (13-d)(9+d)$

(∵ a, b, c are in GP then b^2 = ac)

=>121 = 117 + 4d – d^2 => d^2 −4d + 4 =0

=> $(d-2)^2 = 0$ => d −2 = 0 => d = 2

now 5−2, 5, 5 + 2=> 3, 5, 7

∴ The numbers are 3, 5, 7

17 (a). If the points P(−3, 9) Q(a, b) and R (4, −5)are collinear and a + b = 1, find the values of a and b.

Sol: Given points P (−3, 9) Q(a, b) and R (4, −5) are collinear then

area of triangle PQR = 0

Area of a triangle =

$\frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$=0

$= \frac{1}{2}|-3(b+5) + a(-5-9) + 4(9-b)|$ =0

$= \frac{1}{2}|-3b - 15 - 14a + 36 - 4b| = 0$

$= \frac{1}{2}|-14a - 7b + 21| = 0$

=>2a + b −3 =0 ---- (1) and

given that a + b =1 ----(2)

solving (1) and (2), we get a =2 and b = −1

Which is called fruit sugar?

BIOMOLECULES

Continued from 2nd December..

Monosaccharides

Glucose: Glucose is an aldo hexose and is alsoknown as dextrose because it occurs in nature as the optically active dextro rotatory isomer.

- It is also called grape sugar as it is found in fruits especially grapes contains 20% of Glucose.
- The human blood normally contains 65 to 110mg.of glucose per 100ml.
- In combined form, it occurs in cane sugar and also in polysaccharides such as starch and cellulose.

Preparation: Glucose is prepared in the laboratory by acid hydrolysis of cane sugar in alcoholic solution.

$$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H^+} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

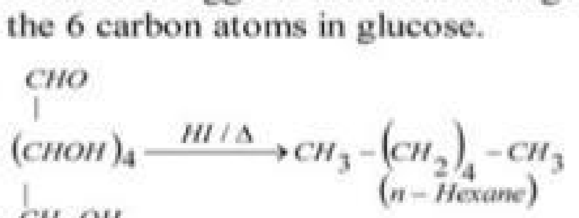
- It is obtained in large scale by the hydrolysis of starch with dil. H_2SO_4 (or) HCl at 2-3 atm pressure & 393 k temp.

$$(C_6H_{10}O_5)_n + nH_2O \xrightarrow{H^+} nC_6H_{12}O_6$$

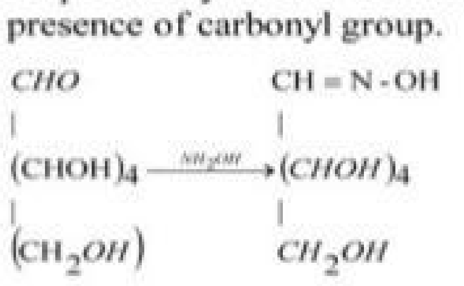
Properties and Sturcutral elucidation

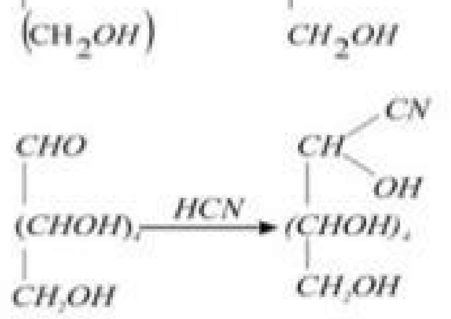
- Molecular formula of glucose is experimentally found as $C_6H_{12}O_6$

- Glucose on prolonged heating with HI gives n-hexane. It suggests the linear arrangement of all the 6 carbon atoms in glucose.

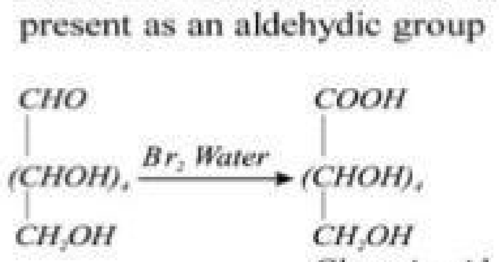


- Glucose reacts with NH_2OH and one molecule of HCN and forms monoxime and cyanohydrin respectively.These reactions suggest the presence of carbonyl group.

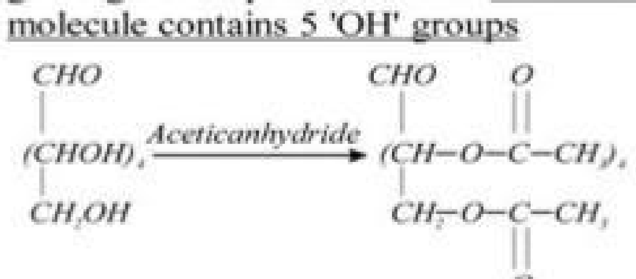




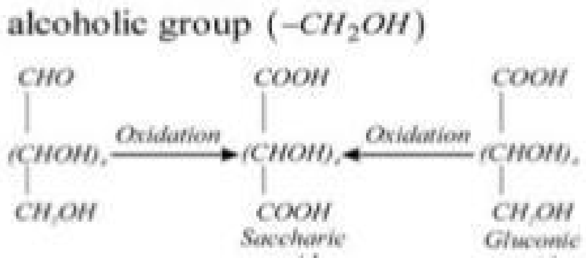
- On reaction with a mild oxidising agent like bromine water, glucose converts to gluconic acid.This indicates that the carbonyl group is present as an aldehydic group



- Glucose reduces Tollen's reagent to metallic silver and also reduces Fehling's solution to reddish brown cuprous oxide and itself gets oxidised to gluconic acid. These reactions suggest that the carbonyl group is an aldehydic group.
- Acylation of Glucose with acetic anhydride gives glucose penta acetate. Hence Glucose molecule contains 5 'OH' groups

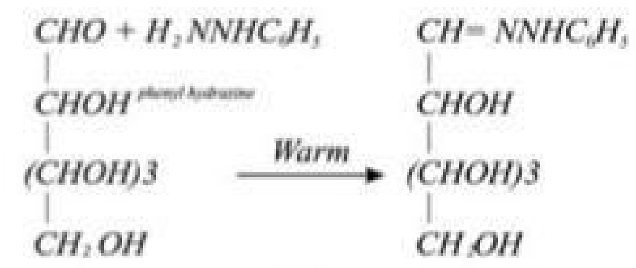


- On oxidation with HNO_3 both glucose and gluconicacid form saccharic acid, a dicarboxylic acid. It suggests the presence of primary alcoholic group $(-CH_2OH)$



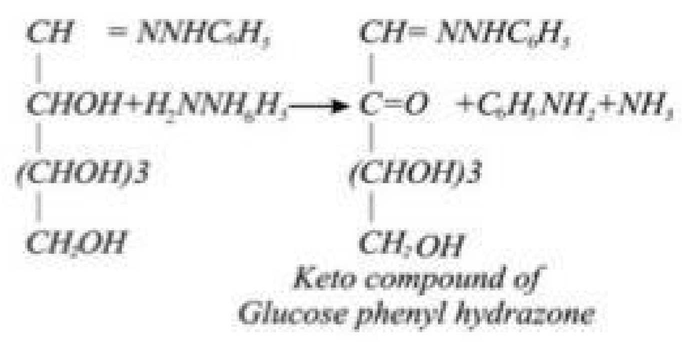
- D-Glucose on reaction with excess of phenyl hydrazine (3 moles of phenyl hydrazine per mole of glucose), forms a dihydrazone known as **osazone**.

Fischer's mechanism : When glucose warmed with excess of phenyl hydrazine, first forms phenylhydrazone by condensation with $-CHO$ group.

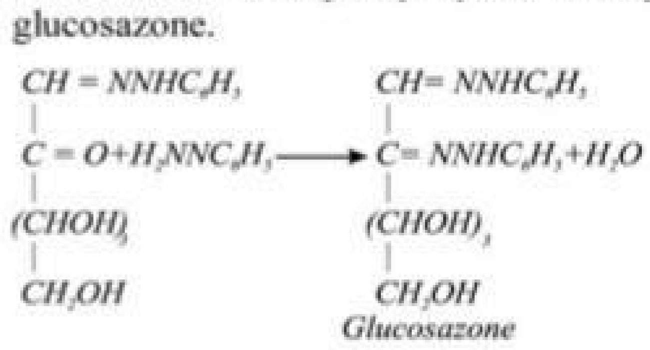


Glucose Glucose Phenyl hydrazone

The adjacent $-CHOH$ group is then oxidised by a second molecule of phenyl hydrazine.



The resulting carbonyl compounds reacts with a third molecule of phenyl hydrazine to yield glucosazone.



Note: All monosaccharides which differ in configuration only at C_1 and C_2 give the same osazone, e.g., D-glucose, D-fructose, D-manose all form the same osazone

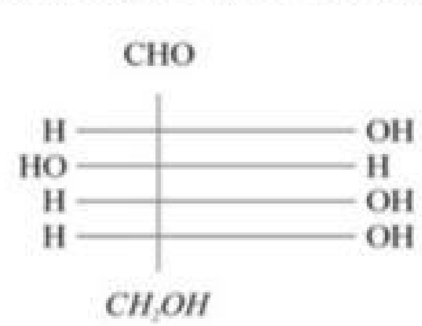
- With conc. NaOH solution, glucose first turns yellow, then brown and finally resinifies.
- With dil. NaOH solution, glucose under goes reversable isomerisation and gives a mixture of D-mannose and D-fructose. This reaction is known as **Lobry de Bruyn-Van Ekenstein rearrangement.**

D-glucose ⇌ D-Manose ⇌ D-Fructose

It is because of this isomerisation that D–fructose reduces Tollens' reagent and Fehling's solution, though fructose does not contain any aldehydic group.

Same results were obtained if manose (or) fructose are treated with alkali. It is concluded that fructose with ketone group also reduces tollen's reagent due to this isomerisation

- **Epimers are a pair of diasteriomers that differ only in the configuration about a single carbon atom.**
  Ex : Glucose and Mannose are C_2 epimers
  D- Iodose and D- Talose $\rightarrow$ c-3 Epimers
  D-Allose and D-gulose $\rightarrow$ c-4 Epimers
  D- Altrose andD-Iode $\rightarrow$ c-4 epimers
- Based on the above properties Glucose has been assigned an open chain D-Glucose by Baeyer.



D-GLUCOSE

Glucose is (2R, 3S, 4R, 5R) – 2, 3, 4, 5, 6 – pentahydroxyhexanal.

Cyclic structure of Glucose: The open chain structure of Glucose proposed by Baeyer explained most of its properties. But it could not explain the following.

- Glucose does not give schiff's test and does not react with $NaHSO_3$ and NH_3, inspite of presence of -CHO group
- Pentacetate of glucose does not react with $-NH_2OH$ group indicating absence of - CHO group.

The aqueous solution of glucose shows mutarotation.

Mutarotation of glucose: When glucose was crystallised from a concentrated solution at 30^0 C, it gives α - form with melting point $146^0 c$ and $[\alpha]_D = +111^0$.

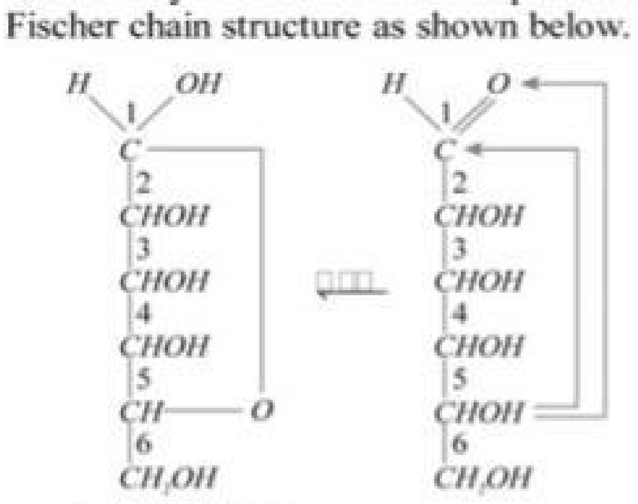
- When glucose crystallised from a hot saturated aqueous solution at a temperature greater than $98^0 C$, gives β-form with a melting point $150^0 C$ and $[\alpha]_D = +19.2^0$.
- These two forms of glucose differ in the stereochemistry at C-1. These two α and β forms, when separately dissolved in water and allowed to stand, their specific rotation gradually change and reach to a specific constant value 52.5^0.
- **This spontaneous change in specific rotations of an optically active compound is called mutarotation..**

$$\underset{[\alpha]_D = +111^o}{\alpha\text{ - D(+)Glucose}} \rightleftharpoons \underset{+52.5^o}{\text{eq.mix}} \rightleftharpoons \underset{+19.2^o}{\beta\text{ - D(+)glucose}}$$

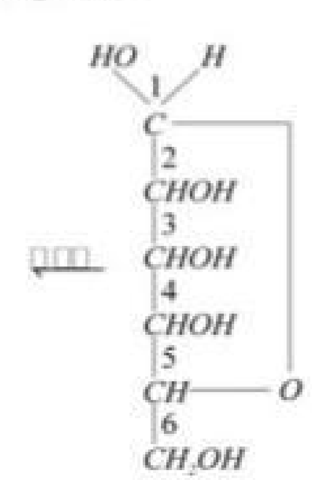
- Equilibrium mixture consists of 36% α-D(+)Glucose and 64% β-D(+)Glucose.
- Above anomalies can be explained by cyclic structure of glucose. Glucose forms a stable cyclic hemiacetal.
- Generally alcoholic groups undergo rapid and reversible addition to aldehyde group to form hemiacetals.
- The alcoholic group bonded to $C-5$ of glucose reacts intramolecularly with $-CHO$ forming a 6-membered hemiacetal ring.
- The asymmetric carbon now at $C-1$ gives two optical isomers. They are not mirror images of each other and hence they are **diastereomers**. They differ in the configuration only at $C-1$ and are called **anomers**.

JEE MAIN Special

- The two cyclic forms exist in equilibrium with Fischer chain structure as shown below.

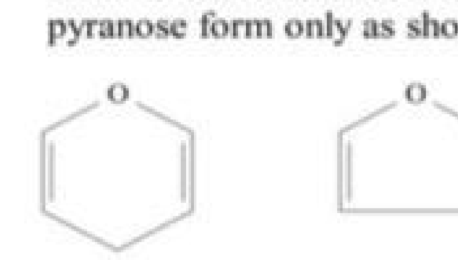


α-D-(+)-glucose



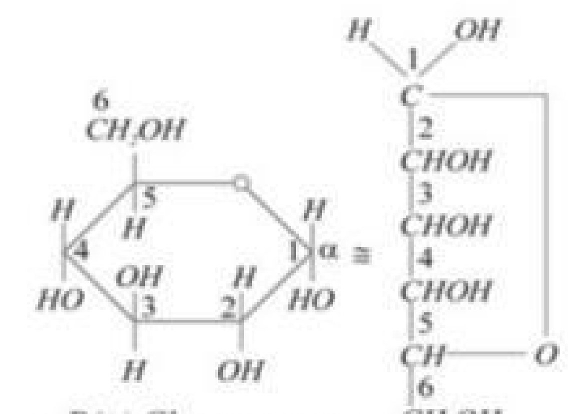
β-D-(+)-glucose

- The α and β forms are confirmed by the reaction of glucose, with methanol in the presence of dry HCl to give methyl α - *D-Glucoside* and methyl β - *D- Glucoside.*
- Glucose forms a six membered ring pyranose containing 5 carbon atoms and one oxygen atom like pyran. The five membered ring formed like furan is called furanose. Glucose is present in pyranose form only as shown in figure.

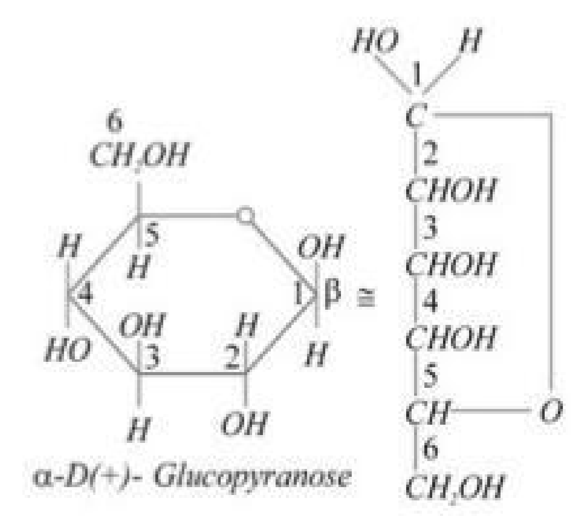


Pyran Furan

- The Haworth horizontal structure of glucopyranose is identical to the Fischer vertical projection structure.
- The groups present on the right side in Fischer formula are written below the plane of the ring and those on the left side are written above the plane.
- The cyclic structure of glucose explains the presence of $\alpha-$ and $\beta-$ forms, mutarotation. It explains the inability of glucose to form aldehyde ammonia and bisulphite compound. In the presence of other carbonyl reagents, the ring is opened and free aldehyde group is produced,



α-D(+)-Glucopyranose



α-D(+)- Glucopyranose

Fructose $(C_6H_{12}O_6)$

- Fructose is a ketohexose. It is also called **Laevulose and fruit sugar**.
- It is laevorotatory compound and belongs to D-series. D-(-) Fructose.
- It is found in ripe fruits and honey.

Preparation

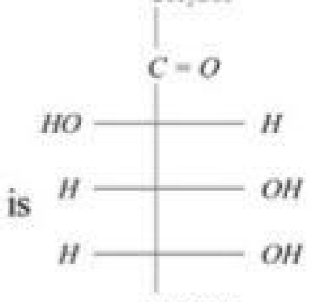
- $C_{12}H_{22}O_{11} + H_2O \rightarrow C_6H_{12}O_6 + C_6H_{12}O_6$
  Sucrose Glucose Fructose
- Like glucose, fructose also shows mutarotation.
- It is reducing sugar.

Structure of Fructose: Fructose contains five hydroxyl groups, out of which two are primary and three are secondary.

- Fructose contains a carbonyl group and it was found to be ketonic from its oxidation products with a strong oxidising agent.
- Fructose was found to contain ketonic functional group at second carbon atom and all the six carbon atoms are in unbranched chain as in the case of glucose.
- Since fructose and glucose form identical osazones when heated with excess of phenyl hydrazine, it was found that both glucose and fructose have the same configuration at $C-3$; $C-4$ and $C-5$.

Though fructose does not contain an aldehydic group, it behaves as reducing sugar due to Lobry de Bruyn van Ekenstein rearrangment.

- Its structure is



- Fructose exists two cyclic forms which are obtained by the addition of -OH at C_5 to the carbonyl group .It is a 5-membered ring and named as furanose ring

To explain all of fructose properties it is suggested with two cyclic structres i.e. $\alpha-D-(-)-$ fructofuranose and $\beta-D-(-)-$ fructofuranose.

$\alpha-$ and $\beta-$ forms of fructose are anomers at $C-2$.

Anomers: Anomers are steroisomers of a cyclic monsaccharide that differ in the position of the OH group at the hemiacetal carbon

Anomers can also be defined as " two sugars that differ in configuration only at the carbon that was the carbonyl carbon in the chain form"

Ex- 1) $\alpha-D$ glucose and $\beta-D$ glucose are anomers
2) $\alpha-D$ fructose and $\beta-D$-fructose are anomers

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