What are Quantum numbers?

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Atomic Structure

- Total number of protons = Atomic number (Z)
- Total number of neutrons n = (A–Z)
- Total number of electrons
  Case-1: for neutral atom e = Z
  Case-2: for cation e = Z – positive charge
  Case-3: for anion e = Z + positive charge
- $\upsilon = \frac{c}{\lambda}$; υ = frequency, c = velocity of light, λ = wavelength.
- Wave number, $\bar{\upsilon} = \frac{1}{\lambda}$
- Quantum energy, $E = h\upsilon = \frac{hc}{\lambda} = hc\bar{\upsilon}$
  where, h = Planck's constant = 6.626×10^{-34} J sec.
- 1 Einstein (Ei) = Energy of ONE MOLE quanta = $Nh\upsilon = Nh\frac{c}{\lambda}$
  where, N = 6.023×10^{23}
- Photoelectric Effect: $h\upsilon = h\upsilon_0 + KE$
- Angular momentum $L = mvr = n\frac{h}{2\pi}$
- Angular momentum = $I\omega = mr^2 \frac{\upsilon}{r} = m\upsilon r$, where
  I = Moment of inertia = mr^2;
  ω = Angular velocity = $\frac{\upsilon}{r}$ where,
  υ = Linear velocity.
- Radius of n^{th} orbit $r = \frac{n^2h^2}{4\pi^2 mKZe^2}$; $r_n = 0.529 \frac{n^2}{Z}$ Å
- Velocity of an electron in n^{th} orbit $= \frac{2\pi Ze^2K}{nh}$; $v_n = 2.18 \quad 10^6 \frac{Z}{n}$ m/sec
- Time period of an electron in its orbit $T = \frac{2\pi r}{v}$, where v = velocity of electron in n^{th} orbit
- Frequency of an electron in its orbit $= \frac{v}{2\pi r}$; r = radius of electron in n^{th} orbit
- Potential energy of electron in a shell $= -\frac{27.2}{n^2} Z^2 eV$
- Kinetic energy of electron in a shell $= +13.6\frac{Z^2}{n^2} eV$
- K.E. $= \frac{KZe^2}{2r}$, P.E. $= -\frac{KZe^2}{r}$, T.E. $= -\frac{KZe^2}{2r}$
- Kinetic energy of electron = – ½ × Potential energy
- Total energy = – K.E.;
- Total energy = $-13.6 \frac{Z^2}{n^2}$ eV per atom, $E_n = -1312 \frac{Z^2}{n^2}$ K J/mol
- Wave number of spectral lines in hydrogen like atoms $= \frac{1}{\lambda} = \bar{\upsilon} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
  $R = 109677 \text{ cm}^{-1} = \frac{13.6eV}{hc}$;

Spectral Series	n_1	n_2	Region	Wavelength
i) Lyman Series	1	2, 3, 4...	UV	$\lambda = \frac{1}{R}\left[\frac{n_2^2}{n_2^2-1}\right]x\frac{1}{z^2}$
ii) Balmer Series	2	3, 4	Visible	$\lambda = \frac{1}{R}\left[\frac{4n_2^2}{n_2^2-4}\right]x\frac{1}{z^2}$
iii) Paschen Series	3	4, 5, 6...	IR	$\lambda = \frac{1}{R}\left[\frac{9n_2^2}{n_2^2-9}\right]x\frac{1}{z^2}$
iv) Brackett Series	4	5, 6, 7 ...	IR	$\lambda = \frac{1}{R}\left[\frac{16n_2^2}{n_2^2-16}\right]x\frac{1}{z^2}$
v) Pfund Series	5	6, 7, ...	IR	$\lambda = \frac{1}{R}\left[\frac{25n_2^2}{n_2^2-25}\right]x\frac{1}{z^2}$
vi) Humphrey Series	6	7, 8	far IR	$\lambda = \frac{1}{R}\left[\frac{36n_2^2}{n_2^2-36}\right]x\frac{1}{z^2}$

- Shortest wavelength spectral line of the series $= \lambda_\infty\left[\frac{n_1^2}{R}\right]x\frac{1}{z^2}$
- Longest wavelength spectral line of the series $\lambda_{long} = \lambda_{first} = \frac{1}{R}\left[\frac{(n_1+1)^2 x n_1^2}{(n_1+1)^2 - n_1^2}\right]x\frac{1}{z^2}$
- Number of photons emitted $= \frac{\Delta n(\Delta n+1)}{2}$,
  where $\Delta n = n_2 - n_1$
  n_1, n_2 are orbit numbers lower and higher respectively.
- In case of single isolated atom if electron make transition from n^{th} state to the ground state then maximum number of spectral lines observed = (n–1)
- Wavelength of a particle moving with a velocity v is $\lambda = \frac{h}{p} = \frac{h}{mv}$
- Wavelength of an electron accelerated with a potential of V is: $\lambda = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2m(K.E.)}}$
- For an electron $\lambda = \left(\frac{150}{V}\right)^{\frac{1}{2}} Å = \frac{12.24}{\sqrt{V}} Å$ [V in volts]
- Circumference of Bohr orbit is equal to the integral multiple of de Broglie wavelength i.e., $2\pi r = n\lambda$.
- Heisenberg's Uncertainty Principle: $\Delta x.\Delta p$ or $x.(mv) \geq \frac{h}{4\pi}$ or $\Delta x.(m\,\Delta v) \geq \frac{h}{4\pi}$
- Wave Mechanical model of atom:
  $\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$

Significance of Quantum numbers

1. Principal quantum number
Symbol: n; Allowed values: 1, 2, 3, 4
Significance: Size and energy of orbit

2. Azimuthal quantum number
Symbol: *l*
Allowed values: 0, 1, 2, ...(n–1)
Significance: Shape of the orbital

3. Magnetic quantum number
Symbol: m
Allowed values: –1, ... 0, ...+1
Significance: Orientation of orbitals in space

4. Spin quantum number
Symbol: s; Allowed values: + ½, –½
Significance: Spin of the electron

- For a given value of n = 3

n	*l*	subshell	orbital
3	0	s	s
	1	p	p_x, p_y, p_z
	2	d	d_{xy}, d_{yz}, d_{xz}, $d_{x^2-y^2}$, d_{z^2}

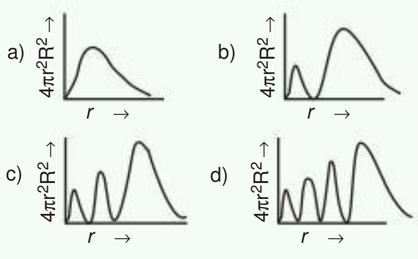
- Maximum electrons in a shell = $2n^2$ (not more than 32)
- Maximum number of orbitals in a shell = n^2
- Maximum number of subshells in a shell = n
- Number of orbitals in a subshell = $2l + 1$
- Maximum number of electrons in particular subshell = $2 \times (2l + 1)$
- Orbital angular momentum L = $L = \sqrt{l(l+1)}\frac{h}{2\pi}$; $L = \sqrt{l(l+1)}\,\hbar$ $\left[\text{dirac } \hbar = \frac{h}{2\pi}\right]$
  for s orbital L = 0, p orbital $L = \sqrt{2}\frac{h}{2\pi}$, d orbital $L = \sqrt{6}\frac{h}{2\pi}$
- * Total spin = Number of unpaired electrons × ½
- * Spin multiplicity = $[2\Sigma s+1]$
- * Spin angular momentum $= \sqrt{s(s+1)}\frac{h}{2\pi}$ where, s = + ½
- * Number of radial nodes or spherical nodes = (n – *l* – 1)
- * Number of nodal planes or angular nodes = *l*
- * Total number of nodes = (n–*l*) excluding node at infinite distance.
- * **Energy of Orbitals:** Energy of electrons in hydrogen atom depends solely on principal quantum number.
  1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f and so on.
  In elements other than hydrogen, orbitals follow following sequence of energy
  1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f ...
- * Half filled and fully filled subshells have extra stability due to greater **exchange energy** and spherical symmetry around the nucleus.
- * Exchange energy $\Delta E = N \times K$
  N = Number of exchanges possible $= \frac{1}{2}\frac{n!}{(n-2)!}$
  where, n = number of electrons having parallel spin.

IPE - Long Answer Questions

1. Write the postulates of Bohr's theory of hydrogen atom? Discuss the importance of this model to explain various series of line spectra in hydrogen atom.
2. How are the quantum numbers n, *l* and m_l arrived? and explain the significance of quantum numbers.
3. a) Explain photo electric effect?
   b) When electromagnetic radiation of wavelength 300 nm falls on the surface of sodium, electrons are emitted with a kinetic energy of 1.68×10^5 J mol^{-1}. What is the minimum energy needed to remove an electron from sodium? What is the maximum wavelength that will cause a photoelectron to be emitted?

JEE Questions

1. A stream of electrons from a heated filment was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron respectively, then the value of h/λ is given by:
   a) $\sqrt{2meV}$ b) $\sqrt{meV}$
   c) 2meV d) meV
2. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesmal thickness, dr, at a distance r from nucleus. The volume of this shell is $4\pi r^2$ dr. The qualitative sketch of dependence of p on r:



Answers: 1) a 2) a

EAMCET Questions

1. The energy of an electron in the 3rd orbit of H-atom(in J) is approximately?
   1) -2.18×10^{-18}
   2) -2.42×10^{-19}
   3) -1.21×10^{-19}
   4) -3.63×10^{-19}
2. The wavelenth(in *m*) of a particle of mass 11.043×10^{-26} kg moving with a velocity of 6.0×10^{-7} ms^{-1}
   1) 1.0×10^{16}
   2) 6.0×10^{-16}
   3) 1.0×10^{-16}
   4) 6.0×10^{16}
3. Using Bohr's equation for the energy levels of the electron in hydrogen atom, determine the energy of an electron in n = 4.
   1) -5.45×10^{-19} J
   2) -1.84×10^{-29} J
   3) -1.36×10^{-19} J
   4) 1.84×10^{-29} J
4. **Assertion (A):** Atoms with completely filled and half filled subshells are stable.
   Reason (R): Completely filled and half filled subshells have symmetrical distribution of electrons and have maximum exchange energy.
   The correct answer is:
   1) (A) and (R) are correct, (R) is the correct explanation of (A)
   2) (A) and (R) are correct, (R) is not the correct explanation of (A)
   3) (A) is correct, but (R) is not correct
   4) (A) is not correct, but (R) is correct
5. The element with the electronic configuration $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^1$ is?
   1) Cu 2) Ca
   3) Cr 4) Co
6. Observe the following table?

Metal	Li	Na	K	Mg	Cu	Ag
Work function/eV	2.42	2.3	2.25	3.7	4.8	4.3

Which are the elements capable of exhibiting photoelectric effect with 295 nm radiation?
1) Li, Na, K and Mg
2) K, Mg, Cu and Mg
3) Na, K, Mg and Cu 4) None

Answers
1) 2 2) 3 3) 3
4) 1 5) 1 6) 1