CHAPTER – 12

ATOMS

Q 12.1 Choose the correct alternative from the clues given at the end of the each statement:

a) The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

b) In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/ Rutherford's model.)

c) A classical atom based on .... is doomed to collapse. (Thomson's model/ Rutherford's model.)

d) An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in .... (Thomson's model/Rutherford's model.)

e) The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models.)

Answer:

a) The size of the atom in Thomson's model is no different from the atomic size in Rutherford's model.

b) In the ground state of Thomson's model electrons are in stable equilibrium, while in Rutherford's model electrons always experience a net force.

c) A classical atom based on Rutherford's model is doomed to collapse.

d) An atom has a nearly continuous mass distribution in a Thomson's model but has a highly non-uniform mass distribution in Rutherford's model.

e) The positively charged part of the atom possesses most of the mass in both the models.

Q 12.2 Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Answer:

On repeating the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil we would have different observations than Rutherford, as the alpha particles won't be scattered much because of being heavier than the nucleus of the Hydrogen atom. Therefore we would not be able to confirm the presence of almost the entire mass of the atom at its centre.

Q 12.3 What is the shortest wavelength present in the Paschen series of spectral lines?

Answer:

The Rydberg's formula for the hydrogen atom is

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where R is Rydberg constant for the Hydrogen atom and equals to 1.1×10^7 m^{-1}

For shortest wavelength in Paschen Series n_1=2 and n_2= ∞

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$$

$$\lambda = 8.18 \times 10^{-7} m$$

The shortest wavelength in Paschen Series is therefore 818 nm.

Q 12.4 A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

Answer:

Frequency of radiation consisting of photons of energy E is given by

$v = \frac{E}{h}$

E=2.3 eV

Plank's constant(h)=6.62×10^{-34} Js

$$v = \frac{2.3 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}}$$

$$v = 5.55 \times 10^{14}\ Hz$$

Q 12.5 The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

Answer:

The ground state energy E=-13.6 eV.

The kinetic energy= -E=13.6 eV

Also ground state energy = Kinetic energy+Potential energy

E=K+U

U=E-K

U=-13.6-13.6

U=-27.2 eV

The kinetic and potential energies are 13.6 eV and -27.2 eV respectively.

Q 12.6 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.

Answer:

The initial energy of the electron is E_1

$$E_1 = -\frac{13.6}{1^2}$$

E_1 =-13.6 eV

The energy of the electron when it is excited to level n=4 is E_2

$$E_2 = -\frac{13.6}{4^2}$$

E_2=-0.85 eV

The difference between these two energy levels is equal to the energy of the photon absorbed by the electron.

The energy of the photon $\Delta E = E_2 - E_1$

ΔE = -0.85 -(-13.6)

ΔE = 12.75 eV

The wavelength of the photon can be calculated using relation

$$\Delta E = \frac{hc}{\lambda}$$

hc=1240 eV

$$\lambda = \frac{hc}{\Delta E}$$

$$\lambda = \frac{1240}{12.57}$$

$$\lambda = 98.6$$

$$v = \frac{c}{\lambda}$$

$$v = \frac{3 \times 10^8}{98.6 \times 10^{-9}}$$

$$v = 3.04 \times 10^{15} Hz$$

The wavelength and frequency of the photon absorbed by the hydrogen atom are 98.6 nm and 3.04×10^{15} Hz respectively.

Q 12.7 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels

Answer:

As per Bohr's model the angular momentum of electrons in each orbit is constant and a multiple of $\frac{nh}{2\pi}$

$m_e v_n r_n = \frac{nh}{2\pi}$ (i)

The electrostatic force of attraction between the electron and the nucleus provides the required centripetal force for the circular motion of the electron.

$\frac{mv_n^2}{rn} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}$ (ii)

Using equation (i) and (ii) we get

$$v_n = \frac{e^2}{2nh\varepsilon_0}$$

$$r_n = \frac{n^2h^2\varepsilon_0}{m_e\pi e^2}$$

$$r_n = \frac{n^2h^2\varepsilon_0}{m_e\pi e^2}$$

$$v_1 = \frac{e^2}{2h\varepsilon_0}$$

$$v_1 = \frac{(1.6\times10^{-19})^2}{2\times6.62\times10^{-34}\times8.85\times10^{-12}}$$

$v_1=2.18\times10^6$ ms^{-1}

$$v_2 = \frac{e^2}{4h\varepsilon_0}$$

$$v_1 = \frac{(1.6\times10^{-19})^2}{4\times6.62\times10^{-34}\times8.85\times10^{-12}}$$

$v_2=1.09\times10^6$ ms^{-1}

$$v_3 = \frac{e^2}{6h\varepsilon_0}$$

$$v_1 = \frac{(1.6\times10^{-19})^2}{6\times6.62\times10^{-34}\times8.85\times10^{-12}}$$

$v_3=7.28\times10^5$ ms^{-1}

Q 12.7 (b) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels

(b) calculate the orbital period in each of these levels.

Answer:

Orbital period (T_n) is defined as time taken by the electron to complete one revolution around the nucleus and is given by

$$T_n = \frac{2\pi r_n}{v_n}$$

$$T_n = \frac{4n^3 h^3 \varepsilon_0^2}{m_e e^4}$$

$$T_1 = \frac{4 \times 1^3 \times (6.62 \times 10^{-34})^2 \times (8.85 \times 10^{-12})^2}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}$$

T_1=1.53×10^{-16} s

$$T_2 = \frac{4 \times 2^3 \times (6.62 \times 10^{-34})^2 \times (8.85 \times 10^{-12})^2}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}$$

T_2=1.22×10^{-15} s

$$T_3 = \frac{4 \times 3^3 \times (6.62 \times 10^{-34})^2 \times (8.85 \times 10^{-12})^2}{9.1 \times 10^{31} \times (1.6 \times 10^{-19})^4}$$

T3=4.12×10^{-15} s

Q 12.8 The radius of the innermost electron orbit of a hydrogen atom is $5.3 \times 10^{-11} m$. What are the radii of the n = 2 and n =3 orbits?

Answer:

The radius of the orbit is proportional to the square of n.

For n=2 the radius of the orbit is

$$r_2 = r_1 \times 2^2$$

$$= 5.3 \times 10^{-11} \times 4$$

$$= 2.12 \times 10^{-10}\ m$$

For n=3 the radius of the orbit is

$$r_3 = r_1 \times 3^2$$

$$= 5.3 \times 10^{-11} \times 9$$

$$= 4.77 \times 10^{-10} m$$

Q 12.9 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Answer:

Since the energy of the electron beam is 12.5 eV the Hydrogen atoms will get excited to all requiring energy equal to or less than 12.5 eV

E_1 =-13.6 eV

E_3 = -1.5 eV

E_3 -E_1 = 12.1 eV

E_4= -0.85 eV

E_4-E_1=12.75 eV

Therefore the electron can reach maximum upto the level n=3.

During de-excitations, the electron can jump directly from n=3 to n=1 or it can first jump from n=3 to n=2 and then from n=2 to n=1

Therefore two wavelengths from the Lyman series and one from the Balmer series will be emitted

To find the wavelengths emitted we will use the Rydberg's Formula

$\frac{1}{\lambda} = R(\frac{1}{n_1^2} - \frac{1}{n_2^2})$ where R is the Rydberg's constant and equals 1.097×10^7 m^{-1}

For n_1=1 and n_2=3

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

Emitted wavelength is 102.5 nm

For n_1=1 and n_2=2

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

Emitted wavelength is 121.54 nm

For n_1=2 and n_2=3

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

Emitted wavelength is 656.3 nm

Q 12.10 In accordance with the Bohr’s model, find the quantum number that characterises the earth’s revolution around the sun in an orbit of radius $1.5 \times 10^{11} m$ m with orbital speed $3 \times 10^4 m/s$ (Mass of earth = $6.0 \times 10^{24}\ kg$.)

Answer:

As per the Bohr's model, the angular of the Earth will be quantized and will be a multiple of $\frac{h}{2\pi}$

$$mvr = \frac{nh}{2\pi}$$

$$n = \frac{2\pi mvr}{h}$$

$$n = \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}}$$

n = 2.56×10^{74}

Therefore the quantum number that characterises the earth's revolution around the sun in an orbit of radius $1.5 \times 10^{11} m$ m with an orbital speed $3 \times 10^{4}\ m/s$

is 2.56×10^{74}

Q 12.11 (a) Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Answer:

The average angle of deflection of α-particles by a thin gold foil predicted by both the models is about the same.

Q 12.11 (b) Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Answer:

The probability of backward scattering predicted by Thomson's model is much less than that predicted by Rutherford's model.

Q 12.11 (c) Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

Answer:

Scattering at moderate angles requires head-on collision the probability of which increases with the number of target atoms in the path of α-particles which increases linearly with the thickness of the gold foil and therefore the linear dependence between the number of α-particles scattered at a moderate angle and the thickness t of the gold foil.

Q 12.11 (d) Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Answer:

It is completely wrong to ignore multiple scattering for the calculation of the average angle of scattering of α-particles by a thin foil in Thomson's model as the deflection caused by a single collision in this model is very small.

Q 12.12 The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Answer:

As per the bohrs model

$m_e v_n r_n = \frac{nh}{2\pi}$ (i)

If the proton and the electron were bound only by the gravity the gravitational force between them will provide the centripetal force required for circular motion

$\frac{m_e v_n^2}{r_n}$ (ii)

From equation (i) and (ii) we can calculate that the radius of the ground state (for n=1) will be

$$r_1 = \frac{h^2}{4\pi G m_p m_e^2}$$

$$r_1 = \frac{(6.62 \times 10^{-34})^2}{4\pi \times 6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times (9.1 \times 10^{-31})^2}$$

$$r_1 \approx 1.2 \times 10^{29}\ m$$

The above value is larger in order than the diameter of the observable universe. This shows how much weak the gravitational forces of attraction as compared to electrostatic forces.

Q 12.13 Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level (n–1). For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Answer:

Using Bohr's model we have.

$$v_n = \frac{e^2}{2nh\varepsilon_0}$$

$$r_n = \frac{n^2 h^2 \varepsilon_0}{m_e \pi e^2}$$

$$E_n = \frac{1}{2} m v_2^2 - \frac{e^2}{4\pi\varepsilon_0 r_n^2}$$

$$E_n = -\frac{me^4}{8n^2h^2\varepsilon_0^2}$$

$$E_n - E_{n-1} = \frac{me^4}{8n^2h^2\varepsilon_0^2} - \frac{me^4}{8(n-1)^2h^2\varepsilon_0^2}$$

$$E_n - E_{n-1} = -\frac{me^4}{8h^2\varepsilon_0^2}\left[\frac{1}{n^2} - \frac{1}{(n-1)^2}\right]$$

$$E_n - E_{n-1} = -\frac{me^4}{8h^2\varepsilon_0^2}\left[\frac{-2n+1}{n^2(n-1)^2}\right]$$

Since n is very large 2n-1 can be taken as 2n and n-1 as n

$$E_n - E_{n-1} = \frac{me^4}{4n^3h^2\varepsilon_0^2}$$

The frequency of the emission caused by de-excitation from n to n-1 would be

$$v = \frac{E_n - E_{n-1}}{h}$$

$$v = \frac{me^4}{4n^3h^3\varepsilon_0^2}$$

The classical frequency of revolution of the electron in the nth orbit is given by

$$v = \frac{v_n}{2\pi r_n}$$

$$v = \frac{e^2}{2nh\varepsilon_0} \times \frac{m_e\pi e^2}{2\pi n^2h^2\varepsilon_0}$$

$$v = \frac{me^4}{4n^3h^3\varepsilon_0^2}$$

The above is the same as the frequency of the emission during de-excitation from n to n-1.

Q 12.14 (a) Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not,say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom ($\approx 10^{-10}m$).

(a) construct a quantity with the dimensions of length from the fundamental constants e, me, and c. Determine its numerical value.

Answer:

Using dimensional analysis we can see that the quantity to be constructed and consisting of m_e, e and c will also have ε_0 and will be equal to

$\frac{e^2}{\varepsilon_0 m_e c^2}$ and has numerical value 3.5×10^{-14} which is much smaller than the order of atomic radii.

Q 12.14 (b) Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom ($\approx 10^{-10} m$).

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, me, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, me, and e and confirm that its numerical value has indeed the correct order of magnitude.

Answer:

Using dimensional analysis we can see that the quantity to be constructed and consisting of m_e, e and h will also have ε_0 and will be equal to

$\frac{\varepsilon_0 h^2}{m_e e^2}$ and has a numerical value of approximately 6.657×10^{-10} which is about the order of atomic radii.

Q 12.15 (a) The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV.

What is the kinetic energy of the electron in this state?

Answer:

Since we know that kinetic energy is equal to the negative of the total energy

K=-E

K=-(-3.4)

K=3.4 eV

Q 12.15 (b) The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV.

What is the potential energy of the electron in this state?

Answer:

Total Energy= Potential energy + Kinetic Energy

E=U+K

U=E-K

U=-3.4-3.4

U=-6.8 eV

Q 12.15 (c) The total energy of an electron in the first excited state of the hydrogen atom is about - 3.4eV.

Which of the answers above would change if the choice of the zero of potential energy is changed?

Answer:

The total energy would change if the choice of the zero of potential energy is changed.

Q 12.16 If Bohr's quantisation postulate (angular momentum = $\frac{nh}{2\pi}$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Answer:

We never speak of Bohr's quantization postulate while studying planetary motion or even motion of other macroscopic objects because they have angular momentum very large relative to the value of h. In fact, their angular momentum is so large as compared to the value of h that the angular momentum of the earth has a quantum number of order 10^{70} . Therefore the angular momentum of such large objects is taken to be continuous rather than quantized.

Q 12.17 Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about 207m_e orbits around a proton].

Answer:

As per Bohr's quantization postulate

$$m_\mu - v_n r_n = \frac{nh}{2\pi}$$

Similarly, like the case in a simple hydrogen atom, the electrostatic force acts centripetally

$$\frac{m_\mu - v_n^2}{r_n} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}$$

From the above relations, we can see that in Bohr's model the Radius is inversely proportional to the mass of the orbiting body and Energy is directly proportional to the mass of the orbiting body.

In case of hydrogen, atom r_1 is 5.3×10^{-11} m

Therefore in case of a muonic hydrogen atom

$$r_1 = \frac{5.3 \times 10^{-11}}{207}$$

$r_1 = 2.56\times10^{-13}$ m

In case of the hydrogen atom, E_1 is -13.6 eV

Therefore in case of a muonic hydrogen atom

E_1=207×(-13.6)

E_1=2.81 keV