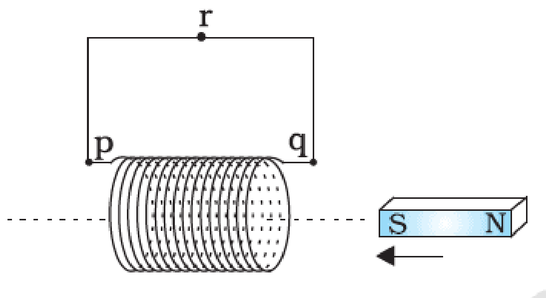
CHAPTER – 6

ELECTROMAGNETIC INDUCTION

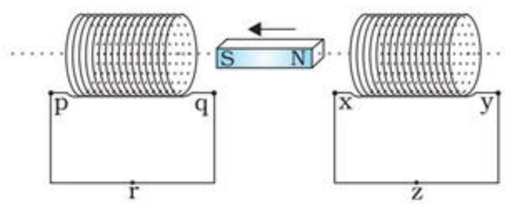
Q 6.1(a) Predict the direction of induced current in the situations described by the following



Answer:

To oppose the magnetic field current should flow in anti-clockwise, so the direction of the induced current is qrpq

Q 6.1 (b) Predict the direction of induced current in the situations described by the following Figs.

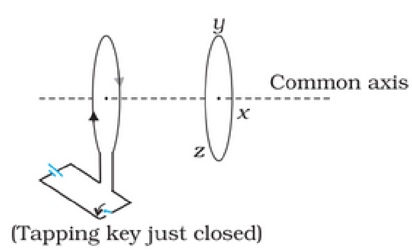


(b)

Answer:

Current in the wire in a way such that it opposes the change in flux through the loop. Here hence current will induce in the direction of p--->r--->q in the first coil and y--->z--->x in the second coil.

Q 6.1 (c) Predict the direction of induced current in the situations described by the following Figs.(c)

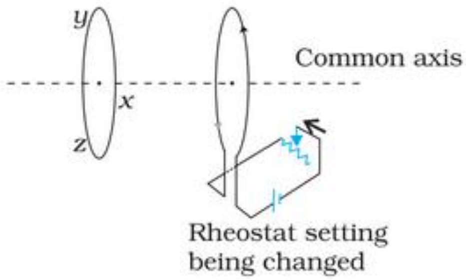


(c)

Answer:

When we close the key, the current will flow through the first loop and suddenly magnetic flux will flow through it such that magnetic rays will go from right to left of the first loop. Now, to oppose this change currently in the second loop will flow such that magnetic rays go from left to right which is the direction yzxy

Q 6.1 (d) Predict the direction of induced current in the situations described by the following Fig. (d)



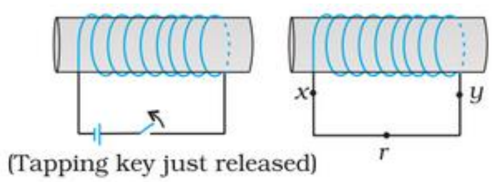
(d)

Answer:

When we increase the resistance of the rheostat, the current will decrease which means flux will decrease so current will be induced to increase the flux through it. Flux will increase if current flows in xyzx.

On the other hand, if we decrease the resistance that will increase the current which means flux will be an increase, so current will induce to reduce the flux. Flux will be reduced if current goes in direction zyxz

Q 6.1 (e) Predict the direction of induced current in the situations described by the following Fig(e)

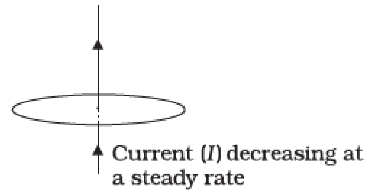


(e)

Answer:

As we release the tapping key current will induce to increase the flux. Flux will increase when current flows in direction xryx.

Q 6.1 (f) Predict the direction of induced current in the situations described by the following Fig (f)



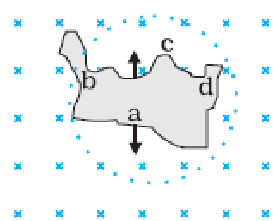
(f)

Answer:

The current will not induce as the magnetic field lines are parallel to the plane. In other words, since flux through the loop is constant (zero in fact), there won't be any induction of the current.

Q6.2 (a) Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19: a

A wire of irregular shape turning into a circular shape;



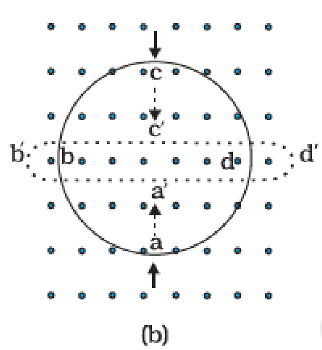
(a)

Answer:

By turning the wire from irregular shape to circle, we are increasing the area of the loop so flux will increase so current will induce in such a way that reduces the flux through it. By right-hand thumb rule direction of current is adcba.

Q6.2 (b) Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19 b :

A circular loop being deformed into a narrow straight wire.



(b)

Answer:

Here, by changing shape, we are decreasing the area or decreasing the flux, so the current will induce in a manner such that it increases the flux. Since the magnetic field is coming out of the plane, the direction of the current will be adcba.

Q6.3 A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0A to 4.0A in 0.1s, what is the induced emf in the loop while the current is changing?

Answer:

Given in a solenoid,

The number of turn per unit length :

$n = 15\, turn/cm = 1500\, turn/m$

loop area :

$A = 2cm^2 = 2 \times 10^{-4}$

Current in the solenoid :

$$initial\ current = I_{initial} = 2$$

$$final\ current = I_{final} = 4$$

change in current :

$\Delta I = 4 - 2 = 2$

change in time:

$$\Delta t = 0.1s$$

Now, the induced emf :

$$e = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = \frac{d(\mu_0 nIA)}{dt} = \mu_0 nA\frac{dI}{dt} = \mu_0 nA\frac{\Delta I}{\Delta t}$$

$e = 4\pi \times 10^{-7} \times 1500 \times 10^{-4} \times \frac{2}{0.1} = 7.54 \times 10^{-6}$

hence induced emf in the loop is 7.54×10^{-6}.

Q6.4 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Answer:

Given:

Length of rectangular loop :

$$l = 8cm = 0.08\, m$$

Width of the rectangular loop:

$$b = 2cm = 0.02m$$

Area of the rectangular loop:

$$A = l \times b = (0.08)(0.02)m^2 = 16 \times 10^{-4}m^2$$

Strength of the magnetic field

$$B = 0.3T$$

The velocity of the loop :

$$v = \frac{1cm}{s} = 0.01\ m/s$$

Now,

a) Induced emf in long side wire of rectangle:

$$e = Blv = 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4}V$$

this emf will be induced till the loop gets out of the magnetic field, so

time for which emf will induce :

$$t = \frac{distance}{velocity} = \frac{b}{v} = \frac{2 \times 10^{-2}}{0.01} = 2s$$

Hence a $2.4 \times 10^{-4}V$ emf will be induced for 2 seconds.

b) Induced emf when we move along the width of the rectangle:

$$e = Bbv = 0.3 \times 0.02 \times 0.01 = 6 \times 10^{-5}V$$

time for which emf will induce :

$$t = \frac{distance}{velocity} = \frac{l}{v} = \frac{8 \times 10^{-2}}{0.01} = 8s$$

Hence a $6 \times 10^{-5}V$ emf will induce for 8 seconds.

Q6.5 A $1.0m$ long metallic rod is rotated with an angular frequency of $400\ rad\ s^{-1}$ about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Answer:

Given

length of metallic rod :

$$l = 1m$$

Angular frequency of rotation :

$$\omega = 400s^{-1}$$

Magnetic field (which is uniform)

$$B = 0.5T$$

Velocity: here velocity at each point of the rod is different. one end of the rod is having zero velocity and another end is having velocity ω r. and hence we take the average velocity of the rod so,

$$Average\ velocity = \frac{0 + \omega l}{2} = \frac{\omega l}{2}$$

Now,

Induce emf

$$e = Blv = Bl\frac{\omega l}{2} = \frac{Bl^2\omega}{2}$$

$$e = \frac{0.5 \times 1^2 \times 400}{2} = 100V$$

Hence emf developed is 100V.

Q6.6 A circular coil of radius $8.0cm$ and $20\ turns$ is rotated about its vertical diameter with an angular speed of $50\ rad\ s^{-1}$ in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω , calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Answer:

Given

The radius of the circular loop $r = 8cm = 0.08m$

Number of turns $N = 20$

Flux through each turn

$$\phi = B.A = B\ A cos\ \theta = BA\ cos\ \omega t = B\pi r^2 \omega t$$

Flux through N turn

$$\phi = NB\pi r^2 cos\ \omega t$$

Induce emf:

$$e = \frac{d\phi}{dt} = \frac{d(N\ B\pi r^2 cos\ \omega t)}{dt} = NBr^2\pi\omega sin\omega$$

Now,

maximum induced emf (when sin function will be maximum)

$$e_{max} = NB\pi r^2\omega = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 0.603V$$

Average induced emf

$e_{average} = 0$ as the average value of sin function is zero,

Maximum current when resistance R of the loop is 10Ω .

$$I_{max} = \frac{e_{max}}{R} = \frac{0.603}{10} = 0.0603A$$

Power loss :

$$P_{loss} = \frac{1}{2}E_0 I_0 = \frac{1}{2}(0.603)(0.0603) = 0.018W$$

Here, power is getting lost as emf is induced and emf is inducing because we are MOVING the conductor in the magnetic field. Hence external force through which we are rotating is the source of this power.

Q6.7 (a) A horizontal straight wire 10m long extending from east to west isfalling with a speed of $50.\, m\, s^{-1}$, at right angles to the horizontal component of the earth's magnetic field, $0.30 \times 10^{-4}\, wb\, m^{-2}$.

What is the instantaneous value of the emf induced in the wire?

Answer:

Given

Length of the wire $l = 10m$

Speed of the wire $v = 5m/s$

The magnetic field of the earth $B = 0.3 \times 10^{-4} Wbm^{-2}$

Now,

The instantaneous value of induced emf :

$$e = Blv = 0.3 \times 10^{-4} \times 10 \times 5 = 1.5 \times 10^{-3}$$

Hence instantaneous emf induce is 1.5×10^{-3}.

Q6.7 (b) A horizontal straight wire 10m long extending from east to west is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, $0.30 \times 10^{-4} wb\, m^{-2}$.

What is the direction of the emf ?

Answer:

If we apply the Flemings right-hand rule, we see that the direction of induced emf is from west to east.

Q6.7(c) A horizontal straight wire 10m long extending from east to west is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, $0.30 \times 10^{-4}\, wb\, m^{-2}$.

Which end of the wire is at the higher electrical potential?

Answer:

The eastern wire will be at the higher potential end.

Q6.8 Current in a circuit falls from $5.0A$ to $0.0A$ in 0.1s . If an average emf of 200V induced, give an estimate of the self-inductance of the circuit.

Answer:

Given

Initial current $I_{initial} = 5A$

Final current $I_{final} = 0A$

Change in time $I_{final} = \Delta t = 0.1s$

Average emf $e = 200V$

Now,

As we know, in an inductor

$$e = L\frac{di}{dt} = L\frac{\Delta I}{\Delta t} = L\frac{I_{final} - I_{initial}}{\Delta t}$$
$$L = \frac{e\Delta t}{I_{final} - I_{initial}} = \frac{200 \times 0.1}{5 - 0} = 4H$$

Hence self-inductance of the circuit is 4H.

Q6.9 A pair of adjacent coils has a mutual inductance of $\mathbf{1.5H}$. If the current in one coil changes from 0 to $\mathbf{20\ A}$ in 0.5 s, what is the change of flux linkage with the other coil?

Answer:

Given

Mutual inductance between two coils:

$$M = 1.5H$$

Currents in a coil:

$$I_{initial} = 0$$
$$I_{final} = 20$$

Change in current:

$di = 20 - 0 = 20$

The time taken for the change

$$dt = 0.5s$$

The relation between emf and mutual inductance:

$$e = M\frac{di}{dt}$$

$$e = \frac{d\phi}{dt} = M\frac{di}{dt}$$

$$d\phi = Mdid\phi = Mdi = 1.5 \times 20 = 30Wb$$

Hence, the change in flux in the coil is 30 Wb.

Q6.10 A jet plane is travelling towards west at a speed of 1800 km/h . What is the voltage difference developed between the ends of the wing having a span of 25m , if the Earth's magnetic field at the location has a magnitude of $5 \times 10^{-4}T$ and the dip angle is 30°.

Answer:

Given

Speed of the plane:

$$v = 1800kmh^{-1} = \frac{1800 \times 1000}{60 \times 60} = 500\, m/s$$

Earth's magnetic field at that location:

$$B = 5 \times 10^{-4}T$$

The angle of dip that is angle made with horizontal by earth magnetic field:

$\delta = 30°$

Length of the wings

$$l = 25\, m$$

Now, Since the only the vertical component of the magnetic field will cut the wings of plane perpendicularly, only those will help in inducing emf.

The vertical component of the earth's magnetic field :

$$B_{vertical} = B\, sin\delta = 5 \times 10^{-4}\, sin\, 30 = 5 \times 10^{-4} \times 0.5 = 2.5 \times 10^{-4}$$

So now, Induce emf :

$$e = B_{vertical}\, lv = 2.5 \times 10^{-4} \times 25 \times 500 = 3.125V$$

Hence voltage difference developed between the ends of the wing is 3.125V.

Q6.11 Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3T at the rate of $0.02\, T\, s^{-1}$. If the cut is joined and the loop has a resistance of 1.6Ω , how much power is dissipated by the loop as heat? What is the source of this power?

Answer:

Given,

Area of the rectangular loop which is held still:

$$A = l \times b = (0.08)(0.02)m^2 = 16 \times 10^{-4}$$

The resistance of the loop:

$$R = 1.6\Omega$$

The initial value of the magnetic field :

$$B_{initial} = 0.3T$$

Rate of decreasing of this magnetic field:

$$\frac{dB}{dt} = 0.02T/s$$

Induced emf in the loop :

$$e = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = 16 \times 10^{-4} \times 0.02 = 0.32 \times 10^{-4}V$$

Induced Current :

$$I_{induced} = \frac{e}{R} = \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5}A$$

The power dissipated in the loop:

$$P = I_{induced}^2 R = (2 \times 10^{-5})^2 \times 1.6 = 6.4 \times 10^{-10}W$$

The external force which is responsible for changing the magnetic field is the actual source of this power.

Q6.12 A square loop of side 12cm with its sides parallel to X and Y axes is moved with a velocity of 8 $cm\,s^{-1}$ in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of $10^{-3}Tcm^{-1}$ along the negative x-direction (that is it increases by $10^{-3}Tcm^{-1}$as one moves in the negative x-direction), and it is decreasing in time at the rate of $10^{-3}Ts^{-1}$. Determine the direction and magnitude of the induced current in the loop if its resistance is $4.50m\Omega$.

Answer:

Given,

Side of the square loop

$$l = 12cm = 0.12m$$

Area of the loop:

$$A = 0.12 \times 0.12m^2 = 144 \times 10^{-4}m^2$$

The resistance of the loop:

$$R = 4.5m\Omega = 4.5 \times 10^{-3}\Omega$$

The velocity of the loop in the positive x-direction

$$v = 8cm/s = 0.08m/s$$

The gradient of the magnetic field in the negative x-direction

$$\frac{dB}{dx} = 10^{-3}\ T/cm\ = 10^{-1}T/m$$

Rate of decrease of magnetic field intensity

$$\frac{dB}{dt} = 10^{-3}\ T/s$$

Now, Here emf is being induced by means of both changing magnetic field with time and changing with space. So let us find out emf induced by both changing of space and time, individually.

Induced emf due to field changing with time:

$$e_{withtime} = \frac{d\phi}{dt} = A\frac{dB}{dt} = 144 \times 10^{-4} \times 10^{-3} = 1.44 \times 10^{-5}\ Tm^2/s$$

Induced emf due to field changing with space:

$$e_{withspace} = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dx}\frac{dx}{dt} = A\frac{dB}{dx}v$$

$$e_{withspace} = 144 \times 10^{4} \times 10^{-1} \times 0.08 = 11.52 \times 10^{-5}\ Tm^2/s$$

Now, Total induced emf :

$$e_{total} = e_{withtime} + e_{withspace} = 1.44 \times 10^{-5} + 11.52 \times 10^{-5} = 12.96 \times 10^{-5}V$$

Total induced current:

$$I = \frac{e}{R} = \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}} = 2.88 \times 10^{-2}A$$

Since the flux is decreasing, the induced current will try to increase the flux through the loop along the positive z-direction.

Q6.13 It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area $2cm^2$ with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5mC . The combined resistance of the coil and the galvanometer is 0.50Ω . Estimate the field strength of magnet.

Answer:

Given,

Area of search coil :

$$A = 2cm^2 = 2 \times 10^{-4}m^2$$

The resistance of coil and galvanometer

$$R = 0.5\Omega$$

The number of turns in the coil:

$$N = 25$$

Charge flowing in the coil

$$Q = 7.5mC = 7.5 \times 10^{-3}C$$

Now.

Induced emf in the search coil

$$e = N\frac{d\phi}{dt} = N\frac{\phi_{final} - \phi_{initial}}{dt} = N\frac{BA - 0}{dt} = \frac{NBA}{dt}$$

$$e = iR = \frac{dQ}{dt}R = \frac{NBA}{dt}$$

$$B = \frac{RdQ}{NA} = \frac{0.5 \times 7.5 \times 10^{-3}}{25 \times 2 \times 10^{-4}} = 0.75T$$

Hence magnetic field strength for the magnet is 0.75T.

Q6.14 (a) Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the $rod = 9.0m\Omega$, $B = 0.50T$, resistance of the closed loop containing the $rod = 9.0m\Omega$. Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of $12cm\ s^{-1}$ in the direction shown. Give the polarity and magnitude of the induced emf.

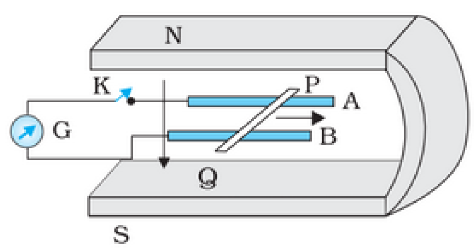


FIGURE 6.20

Answer:

Given

Length of the rod

$l = 15cm = 0.15m$

Speed of the rod

$v = 12\ cm/s = 0.12\ m/s$

Strength of the magnetic field

$$B = 0.5T$$

induced emf in the rod

$$e = Bvl = 0.5 \times 0.12 \times 0.15 = 9 \times 10^{-3}V$$

Hence 9mV emf is induced and it is induced in a way such that P is positive and Q is negative.

Q6.14 (b) Figure 6.20 shows a metal rod $\boldsymbol{PQ}$ resting on the smooth rails $\boldsymbol{AB}$ and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer $\boldsymbol{G}$ connects the rails through a switch $\boldsymbol{K}$. Length of the $\boldsymbol{rod = 15cm}$,$\boldsymbol{B = 0.50T}$, resistance of the closed loop containing the $\boldsymbol{rod = 9.0m\Omega}$. Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of $\boldsymbol{12cm\ s^{-1}}$ in the direction shown. Give the polarity and magnitude of the induced emf.

(b)Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

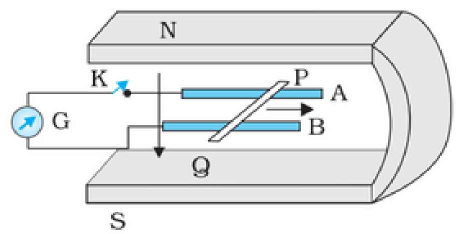


FIGURE 6.20

Answer:

Yes, there will be excess charge built up at the end of the rod when the key is open. This is because when we move the conductor in a magnetic field, the positive and negative charge particles will experience the force and move into the corners.

When we close the key these charged particles start moving in the closed loop and continuous current starts flowing.

Q6.14 (c) Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the $rod = 15\ cm$, $B = 0.50T$, resistance of the closed loop containing the $rod = 9.0m\Omega$. Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of $12\ cm\ s^{-1}$ in the direction shown. Give the polarity and magnitude of the induced emf.

(c) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

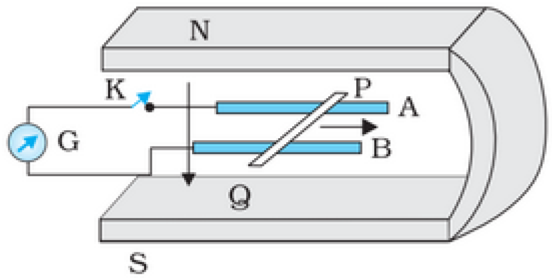


FIGURE 6.20

Answer:

When the key K is open there is excess charge at both ends of the rod. this charged particle creates an electric field between both ends. This electric field exerts electrostatic force in the charged particles which cancel out the force due to magnetic force. That's why net force on a charged particle, in this case, is zero.

Q6.14 (d) Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the $rod = 15cm$, $B = 0.50T$, resistance of the closed loop containing the $rod = 9.0m\Omega$. Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of 12 $cm\ s^{-1}$ in the direction shown. Give the polarity and magnitude of the induced emf.

(d) What is the retarding force on the rod when K is closed?

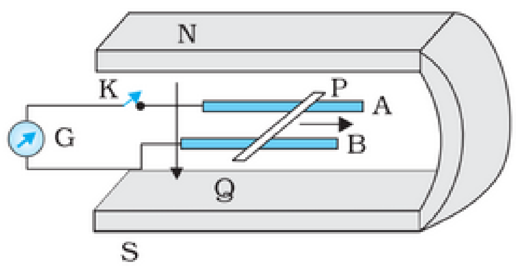


FIGURE 6.20

Answer:

Induced emf = 9mV (calculated in a part of this question)

The resistance of loop with rod = 9mΩ

Induced Current

$$i = \frac{e}{R} = \frac{9mV}{9m\Omega} = 1A$$

Now,

Force on the rod

$$F = Bil = 0.5 \times 1 \times 0.15 = 7.5 \times 10^{-2}$$

Hence retarding force when k is closed is 7.5×10^{-2}.

Q6.14 (e) Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the $\boldsymbol{rod} = \mathbf{15\ cm}$, $\boldsymbol{B} = \mathbf{0.50}\boldsymbol{T}$, resistance of the closed loop containing the $\boldsymbol{rod} =$ $\mathbf{9.0}\boldsymbol{m}$□ . Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of $\mathbf{12}\ \boldsymbol{cm\ s^{-1}}$ in the direction shown. Give the polarity and magnitude of the induced emf.

(e) How much power is required (by an external agent) to keep the rod moving at the same speed $(= 12\ cm\ s^{-1})$ when Kis closed? How much power is required when K is open?

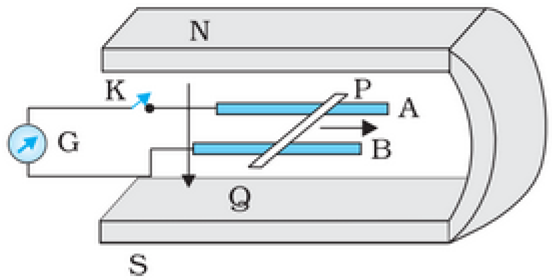


FIGURE 6.20

Answer:

Force on the rod

$$F = 7.5 \times 10^{-2}$$

Speed of the rod

$$v = 12cm/s = 0.12m/s$$

Power required to keep moving the rod at the same speed

$$P = Fv = 7.5 \times 10^{-2} \times 0.12 = 9 \times 10^{-3} = 9mW$$

Hence required power is 9mW.

When the key is open, no power is required to keep moving rod at the same speed.

Q6.14 (f) Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the $\boldsymbol{rod} = \mathbf{15}\boldsymbol{cm}$,$\boldsymbol{B} = \mathbf{0.50}\boldsymbol{T}$, resistance of the closed loop containing the $\boldsymbol{rod} =$ $\mathbf{9.0}\boldsymbol{m}$□ . Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of $\mathbf{12}\boldsymbol{cm\ s^{-1}}$ in the direction shown. Give the polarity and magnitude of the induced emf

(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?

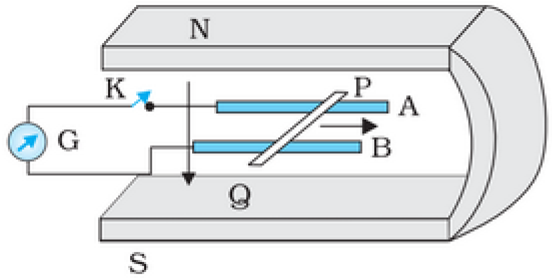


FIGURE 6.20

Answer:

Current in the circuit i = 1A

The resistance of the circuit R = 9mΩ

The power which is dissipated as the heat

$$P_{heat} = i^2R = 1^2 \times 9 \times 10^{-3} = 9mW$$

Hence 9mW of heat is dissipated.

We are moving the rod which induces the current. The external agent through which we are moving our rod is the source of the power.

Q6.14 (g) Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the $rod = 15\ cm$, $B = 0.50T$, resistance of the closed loop containing the $rod = 9.0m\Omega$. Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of 12 ms^{-1} in the direction shown. Give the polarity and magnitude of the induced emf

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

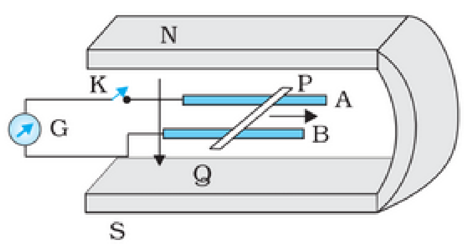


FIGURE 6.20

Answer:

If the magnetic field is parallel to the rail then, the motion of the rod will not cut across the magnetic field lines and hence no emf will induce. Hence emf induced is zero in this case.

Q6.15 An air-cored solenoid with length $30cm$, area of cross-section $25cm^2$ and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of

$10^{-3}s$. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Answer:

Given

Length of the solenoid $l = 30cm = 0.3m$

Area of the cross-section of the solenoid $A = 25cm^2 = 25 \times 10^{-4}m^2$

Number of turns in the solenoid $N = 500$

Current flowing in the solenoid $I = 2.5A$

The time interval for which current flows $\Delta t = 10^{-3}s$

Now.

Initial flux:

$$\phi_{initial} = NBA = N\left(\frac{\mu_0 NI}{l}\right)A = \frac{\mu_0 N^2 IA}{l}$$

$$\phi_{initial} = \frac{4\pi \times 10^{-7} \times 500^2 \times 2.5 \times 25 \times 10^{-4}}{0.3} = 6.55 \times 10^{-3}Wb$$

Final flux: since no current is flowing,

$$\phi_{final} = 0$$

Now

Induced emf:

$$e = \frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t} = \frac{\phi_{final} - \phi_{initial}}{\Delta t} = \frac{6.55 \times 10^{-3} - 0}{10^{-3}} = 6.55V$$

Hence 6.55V of average back emf is induced.

Q6.16 (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig.

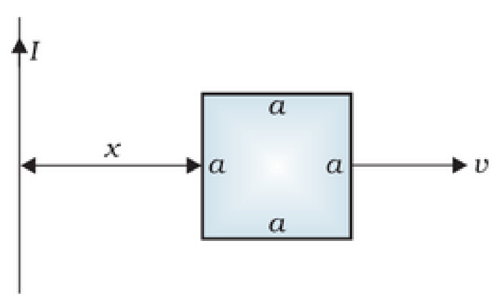


FIGURE 6.21

Answer:

Here let's take a small element dy in the loop at y distance from the wire

Area of this element dy :

$$dA = a \times dy$$

The magnetic field at dy (which is y distance away from the wire)

$$B = \frac{\mu_0 I}{2\pi y}$$

The magnetic field associated with this element dy

$$\phi = BdA$$

$$d\phi = \frac{\mu_0 I}{2\pi y} \times ady = \frac{\mu_0 I\ a}{2\pi}\frac{dy}{y}$$

$$\phi = \int_x^{a+x} \frac{\mu_0 Ia}{2\pi}\frac{dy}{y} = \frac{\mu_0 Ia}{2\pi}[lnx]_a^{a+x} = \frac{\mu_0 Ia}{2\pi} ln\left[\frac{a+x}{x}\right]$$

Now As we know

$\phi = MI$ where M is the mutual inductance

so

$$\phi = MI = \frac{\mu_0 Ia}{2\pi}\ ln\ \left[\frac{a+x}{x}\right]$$

$$M = \frac{\mu_0 Ia}{2\pi} ln\ \left[\frac{a+x}{x}\right]$$

Hence mutual inductance between the wire and the loop is:

$$\frac{\mu_0 a}{2\pi}\ ln\ \left[\frac{a+x}{x}\right]$$

Q6.16 (b) Now assume that the straight wire carries a current of 50A and the loop is moved to the right with a constant velocity, V = 10 m/s. Calculate the induced emf in the loop at the instant when X=0.2m. Take a = 0.1m and assume that the loop has a large resistance.

Answer:

Given,

Current in the straight wire

I= 50A

Speed of the Loop which is moving in the right direction

$V = 10m/s$

Length of the square loop

$$a = 0.1m$$

distance from the wire to the left side of the square

$$X = 0.2m$$

Now,

Induced emf in the loop :

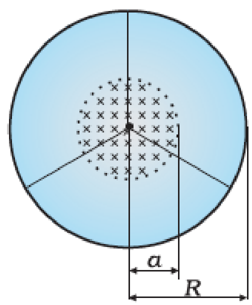
$$E = B_x av = \frac{\mu_0 I}{2\pi x} av = \left(\frac{4\pi \times 10^{-7} \times 50}{2\pi \times 0.2}\right) \times 0.1 \times 10 = 5 \times 10^{-5} V$$

Hence emf induced is $5 \times 10^{-5} V$.

6.17) A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,

$$B = -B_0 K \;\; (r \le a; a < R) = 0 \;\; (otherwise)$$

What is the angular velocity of the wheel after the field is suddenly switched off?



Answer:

Given,

The radius of the wheel =R

The mass of the wheel = M

Line charge per unit length when the total charge is Q

$$\lambda = \frac{Q}{2\pi r}$$

Magnetic field :

$$B = -B_0 K \;\; (r \le a; a < R) = 0$$

Magnetic force is balanced by centrifugal force when v is the speed of the wheel that is

$$BQv = \frac{mv^2}{r}$$

$$B2\pi r\lambda = \frac{mv}{r}$$

$$v = \frac{B2\pi\lambda r^2}{M}$$

Angular velocity of the wheel

$$w = -\frac{B2\pi r\lambda a^2}{MR}$$

when $(r \leq a; a < R)$

$$w = -\frac{B2\pi\lambda a^2}{MR}$$

It is the angular velocity of the wheel when the field is suddenly shut off.