# CHAPTER – 13

# NUCLEI

**Q.13.1 (a)** Two stable isotopes of lithium ${}^6_3Li$ and ${}^7_3Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses $6.01512\ u$ and $7.01600\ u$, respectively. Find the atomic mass of lithium.

**Answer:**

Mass of the two stable isotopes and their respective abundances are $6.01512\ u$ and $7.01600\ u$ and 7.5% and 92.5%.

$$m = \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{100}$$

m=6.940934 u

**Q. 13.1(b)** Boron has two stable isotopes, ${}^{10}_5B$ and ${}^{11}_5B$. Their respective masses are $10.01294\ u$ and $11.00931\ u$, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_5B$ and ${}^{11}_5B$.

**Answer:**

The atomic mass of boron is 10.811 u

Mass of the two stable isotopes are $10.01294\ u$ and $11.00931\ u$ respectively

Let the two isotopes have abundances x% and (100-x)%

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{100}$$

Therefore the abundance of ${}^{10}_5B$ is 19.89% and that of ${}^{11}_5B$ is 80.11%

**Q. 13.2** The three stable isotopes of neon: ${}^{20}_{10}Ne{}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are $19.99\ u, 20.99\ u, and\ 21.99\ u,$ respectively. Obtain the average atomic mass of neon.

**Answer:**

The atomic masses of the three isotopes are 19.99 u($m_1$), 20.99 u($m_2$) and 21.99u($m_3$)

Their respective abundances are 90.51%($p_1$), 0.27%($p_2$) and 9.22%($p_3$)

$$m = \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{100}$$

$$m = 20.1771\ u$$

The average atomic mass of neon is 20.1771 u.

**Q. 13.3** Obtain the binding energy( in MeV ) of a nitrogen nucleus $({}^{14}_7N)$, given $m\ ({}^{14}_7N) = 14.00307\ u$

**Answer:**

$m_n$ = 1.00866 u

$m_p$= 1.00727 u

Atomic mass of Nitrogen m= 14.00307 u

Mass defect $\Delta m=7\times m_n+7\times m_p$ - m

$\Delta m=7\times1.00866+7\times1.00727 - 14.00307$

$\Delta m=0.10844$

Now 1u is equivalent to 931.5 MeV

$E_b=0.10844\times931.5$

$E_b$=101.01186 MeV

Therefore binding energy of a Nitrogen nucleus is 101.01186 MeV.

**Q. 13.4 (i)** Obtain the binding energy of the nuclei ${}^{56}_{26}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data:

(i) $m({}^{56}_{26}Fe) = 55.934939\ u$

**Answer:**

$m_H$ = 1.007825 u

$m_n$ = 1.008665 u

The atomic mass of ${}^{56}_{26}Fe$ is m=55.934939 u

Mass defect

$$\Delta m = (56-26)\times m_H + 26\times m_p - m$$

$$\Delta m = 30\times 1.008665 + 26\times 1.007825 - 55.934939$$

$\Delta m$=0.528461

Now 1u is equivalent to 931.5 MeV

$E_b=0.528461\times931.5$

$E_b$=492.2614215 MeV

Therefore the binding energy of a ${}^{56}_{26}Fe$ nucleus is 492.2614215 MeV.

Average binding energy

$$= \frac{492.26}{56} MeV = 8.79\ MeV$$

**Q. 13.4 (ii)** Obtain the binding energy of the nuclei ${}^{56}_{26}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data:

(ii) $m({}^{209}_{83}Bi) = 2.8.980388\ u$

**Answer:**

$m_H$ = 1.007825 u

$m_n$ = 1.008665 u

The atomic mass of ${}^{209}_{83}Bi$ is m=208.980388 u

Mass defect

$$\Delta m = (209 - 83) + 83 \times m_H - m$$

Δm=126×1.008665+83×1.007825 - 208.980388

Δm=1.760877 u

Now 1u is equivalent to 931.5 MeV

$E_b$=1.760877 × 931.5

$E_b$=1640.2569255 MeV

Therefore the binding energy of a ${}^{209}_{83}Bi$ nucleus is 1640.2569255 MeV.

$$Average\ binding\ \ energy = \frac{1640.25}{208.98} = 7.84\ MeV$$

**Q.13.5** A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}Cu$ atoms (of mass $62.92960\ u$).

**Answer:**

Mass of the coin is w = 3g

Total number of Cu atoms in the coin is n

$$n = \frac{w \times N_A}{Atomic\ Mass}$$

$$n = \frac{3 \times 6.023 \times 10^{23}}{62.92960}$$

n=2.871×$10^{22}$

$m_H$ = 1.007825 u

$m_n$ = 1.008665 u

Atomic mass of ${}^{63}_{29}Cu$ is m=62.92960 u

Mass defect Δm=(63-29)×$m_n$+29×$m_H$ - m

Δm=34×1.008665+29×1.007825 - 62.92960

Δm=0.591935 u

Now 1u is equivalent to 931.5 MeV

$E_b$=0.591935×931.5

$E_b$=551.38745 MeV

Therefore binding energy of a ${}_{29}^{63}Cu$ nucleus is 551.38745 MeV.

The nuclear energy that would be required to separate all the neutrons and protons from each other is

$n\times E_b=2.871\times10^{22}\times551.38745$

$=1.5832\times10^{25}$ MeV

$=1.5832\times10^{25}\times1.6\times10^{-19}\times10^{6}$J

$=2.5331\times10^{9}$ kJ

**Q.13.6 (i)** Write nuclear reaction equations for

(i) $\alpha - decay\ of\ {}_{88}^{226}Ra$

**Answer:**

The nuclear reaction equations for the given alpha decay

$${}_{88}^{226}Ra \rightarrow {}_{86}^{222}Rn + {}_{2}^{4}He$$

**Q.13.6 (ii)** Write nuclear reaction equations for

(ii) $\alpha - decay\ of\ {}_{94}^{242}Pu$

**Answer:**

The nuclear reaction equations for the given alpha decay is

$${}_{94}^{242}Pu \rightarrow {}_{92}^{238}U + {}_{2}^{4}He$$

**Q.13.6 (iii)** Write nuclear reaction equations for

(iii) $\beta$*(iii)* $\beta$*(iii)* $\beta^{-} - decay\ of\ {}_{15}^{32}P$

**Answer:**

The nuclear reaction equations for the given beta minus decay is

$${}_{15}^{32}P \rightarrow {}_{16}^{32}S + e^{-} + \bar{\nu}$$

**Q.13.6 (iv)** Write nuclear reaction equations for

(iv) $\beta^{-} -\ decay\ of\ {}_{83}^{210}Bi$

**Answer:**

The nuclear reaction equation for the given beta minus decay is

$${}_{83}^{210}Bi \rightarrow {}_{84}^{210}Po + e^{-} + \bar{\nu}$$

**Q.13.6 (v)** Write nuclear reaction equations for

(v) $\beta^{+} -\ decay\ of\ {}_{6}^{11}C$

**Answer:**

The nuclear reaction for the given beta plus decay will be

$$ {}^{11}_{6}C \rightarrow {}^{11}_{5}P + e^{+} + v $$

**Q.13.6 (vi)** Write nuclear reaction equations for

(vi) $\beta^{+} -\ decay\ of\ {}^{97}_{43}Tc$

**Answer:**

nuclear reaction equations for

$$ \beta^{+} -\ decay\ of\ {}^{97}_{43}Tc\ is $$

$$ {}^{96}_{43}Tc \rightarrow {}^{97}_{42}Mo + e^{+} + v $$

**Q.13.6 (vii)** Write nuclear reaction equations for

Electron capture of ${}^{120}_{54}Xe$

**Answer:**

The nuclear reaction for electron capture of ${}^{120}_{54}Xe$ is

$$ {}^{120}_{54}Xe + e^{-} \rightarrow {}^{120}_{53}I + v $$

**Q. 13.7** A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

**Answer:**

(a) The activity is proportional to the number of radioactive isotopes present

The number of half years in which the number of radioactive isotopes reduces to x% of its original value is n.

$$ n = log_2\left(\frac{100}{x}\right) $$

In this case

$$ n = log_2\left(\frac{100}{3.125}\right) = log_2 32 = 5 $$

It will take 5T years to reach 3.125% of the original activity.

(b) In this case

$$ n = log_2\left(\frac{100}{1}\right) = log_2 100 = 6.64 $$

It will take 6.64T years to reach 1% of the original activity.

**Q.13.8** The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$ , and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

**Answer:**

Since we know that activity is proportional to the number of radioactive isotopes present in the sample.

$$\frac{R}{R_0} = \frac{N}{N_0} = \frac{9}{15} = 0.6$$

Also

$$N = N_0 e^{-\lambda t}$$

$$t = \frac{1}{\lambda} ln \frac{N}{N_0}$$

$$t = -\frac{1}{\lambda} ln\ 0.6$$

$$t = \frac{0.51}{\lambda}$$

but $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$

Therefore

$$t = 0.51 \times \frac{T_{\frac{1}{2}}}{0.693}$$

$$t = 0.735 T_{\frac{1}{2}}$$

$$t \approx 4217$$

The age of the Indus-Valley civilisation calculated using the given specimen is approximately 4217 years.

**Q.13.9** Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

**Answer:**

Required activity=8.0 mCi

1 Ci=$3.7\times10^{10}$ decay $s^{-1}$

8.0 mCi=$8\times10^{-3}\times3.7\times10^{10}$ =$2.96\times10^{8}$ decay $s^{-1}$

$T_{1/2}$=5.3 years

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}}$$

$$\lambda = \frac{0.693}{5.3 \times 365 \times 24 \times 3600}$$

$$\lambda = 4.14 \times 10^{-9}\ s^{-1}$$

$$\frac{dN}{dt} = -N\lambda$$

$$N = -\frac{dN}{dt} \times \frac{1}{\lambda}$$

$$N = -(-2.96 \times 10^{8}) \times \frac{1}{4.14 \times 10^{-9}}$$

$$N = 7.15 \times 10^{16}\ atoms$$

Mass of those many atoms of Cu will be

$$w = \frac{7.15 \times 10^{16} \times 60}{6.023 \times 10^{23}}$$

$$w = 712 \times 10^{-6}\ g$$

$7.12\times10^{-6}$ g of ${}^{60}_{27}Co$ is necessary to provide a radioactive source of 8.0 mCi strength.

**Q. 13.10** The half-life of ${}^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

**Answer:**

$T_{1/2}$=28 years

$$\lambda = \frac{0.693}{28 \times 365 \times 24 \times 3600}$$

$$\lambda = 7.85 \times 10^{-10}\ decay\ s^{-1}$$

The number of atoms in 15 mg of ${}^{90}_{38}Sr$ is

$$N = \frac{15 \times 10^{-3} \times 6.023 \times 10^{23}}{90}$$

$N=1.0038\times10^{20}$

The disintegration rate will be

$$\frac{dN}{dt} = -NA$$

$=-1.0038\times10^{20}\times7.85\times10^{-10}$

$=-7.88\times10^{10}\ s^{-1}$

The disintegration rate is therefore $7.88\times10^{10}$ decay $s^{-1}$.

**Q.13.11** Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}Au$ and the silver isotope ${}^{107}_{47}Ag$

**Answer:**

The nuclear radii are directly proportional to the cube root of the mass number.

The ratio of the radii of the given isotopes is therefore

$$\left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.23$$

**Q.13.12** Find the Q-value and the kinetic energy of the emitted α -particle in the α-decay of

$$(a)\,^{226}_{88}Ra$$

Given $m(^{226}_{88}Ra) = 226.02540\ u,\ \ m(^{222}_{86}Rn) = 222.01750\ u,$

$m(^{222}_{86}Rn) = 220.01137\ u \quad m(^{216}_{84}Po) = 216.00189\ u$

**Answer:**

Mass defect is Δm

$$\Delta m = m(^{226}_{88}Ra) - m(^{222}_{86}Rn) - m(^{4}_{2}He)$$

Δm=226.02540-222.0175-4.002603

Δm=0.005297 u

1 u = 931.5 MeV/$c^2$

Q-value=Δm×931.5

=4.934515 MeV

By using Linear Momentum Conservation and Energy Conservation

The kinetic energy of alpha particle =

$$\frac{mass\ of\ nucleus\ after\ decay}{mass\ of\ nucleus\ before\ decay} \times Q - value$$

$$= \frac{222.01750}{226.0254} \times 4.934515$$

=4.847 MeV

**Q.13.12 (b)** Find the Q-value and the kinetic energy of the emitted α -particle in the α-decay of

$$(b)\,^{220}_{86}Rn$$

Given $m(^{226}_{88}Ra) = 226.02540\ u, \quad m(^{222}_{86}Rn) = 222.01750\ u$

$m(^{222}_{86}Rn) = 220.01137\ u, \quad m(^{216}_{84}Po) = 216.00189\ u,$

**Answer:**

Mass defect is Δm

$$\Delta m = m(^{222}_{86}Rn) - m(^{216}_{84}Po) - m(^{4}_{2}He)$$

Δm=220.01137-216.00189-4.002603

Δm=0.006877 u

1 u = 931.5 MeV/c$^2$

Q-value=Δm×931.5

=6.406 MeV

By using Linear Momentum Conservation and Energy Conservation

The kinetic energy of alpha particle =

$$\frac{mass\ of\ nucleus\ after\ decay}{mass\ of\ nucleaus\ before\ decay} \times Q - value$$

$$= \frac{216.00189}{220.01138} \times 6.406$$

=6.289 MeV

**Q.13.13** The radionuclide $^{11}C$ decays according to

$$^{11}_{6}C \rightarrow B + e^{+} + v;\ \ T_{\frac{1}{2}} = 20.3\ min$$

The maximum energy of the emitted positron is $0.960\ MeV$.

Given the mass values:

$$m(^{11}_{6}C) = 11.011434\ u\ \ and\ \ m(^{11}_{6}B) = 11.009305\ u$$

calculate Q and compare it with the maximum energy of the positron emitted.

**Answer:**

If we use atomic masses

$$\Delta m = m(^{11}_{6}C) - m(^{11}_{5}B) - 2m_e$$

$$\Delta m = 11.011434 - 11.009305 - 2 \times 0.00548$$

$$\Delta m = 0.001033u$$

Q-value= 0.001033×931.5=0.9622 MeV which is comparable with a maximum energy of the emitted positron.

**Q.13.14** The nucleus $^{23}_{10}Ne$ decays by $\beta^{-}$ emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$$(i)\ m(^{23}_{10}Ne) = 22.994466$$

$(ii) m(^{23}_{11}Na) = 22.089770\ u$

**Answer:**

The β decay equation is

$$^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + e^{-} + \bar{v} + Q$$

$$\Delta m = m({}^{23}_{10}Ne) - {}^{23}_{11}Na - m_e$$

$$\Delta m = 22.994466 - 22.989770$$

$$\Delta m = 0.00469\ u$$

(we did not subtract the mass of the electron as it is cancelled because of the presence of one more electron in the sodium atom)

Q=0.004696×931.5

Q=4.3743 eV

The emitted nucleus is way heavier than the β particle and the energy of the antineutrino is also negligible and therefore the maximum energy of the emitted electron is equal to the Q value.

**Q. 13.15 (i)** The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = [m_A + m_b - m_c - m_d]c^2$where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

$(i){}^1_1H + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$ the following

Atomic masses are given to be

$$m({}^2_1H) = 2.014102\ u$$

$$m({}^3_1H) = 3.0016049$$

$$m({}^{12}_6H) = 12.000000\ u$$

$$m({}^{20}_{10}Ne) = 19.992439\ u$$

**Answer:**

$$\Delta m = m({}^1_1H) + m({}^3_1H) - 2m({}^2_1H)$$

$$\Delta m = 1.007825 + 3.001649 - 2 \times 2.014102$$

$$\Delta m = -0.00433$$

The above negative value of mass defect implies there will be a negative Q value and therefore the reaction is endothermic

**Q. 13.15 (ii)** The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by $Q = [m_A + m_b - m_c - m_d]c^2$where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

$$(ii)\ {}^{12}_6C + {}^{12}_6C \rightarrow {}^{20}_{10}Ne + {}^4_2He$$

Atomic masses are given to be

$$m({}^2_1H) = 2.014102\ u$$

$$m({}^3_1H) = 3.0016049\ u$$

$$m({}^{12}_6H) = 12.000000\ u$$

$$m(^{20}_{10}Ne) = 19.992439\,u$$

**Answer:**

$$\Delta m = 2m(^{12}_{6}C) - m(^{20}_{10}Ne) - m(^{4}_{2}He)$$

$$\Delta m = 2 \times 12.00000 - 19.992439 - 4.002603$$

$$\Delta m = 0.004958$$

The above positive value of mass defect implies Q value would be positive and therefore the reaction is exothermic

**Q.13.16** Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$ . Is the fission energetically possible? Argue by working out Q of the process. Given $m(^{56}_{26}Fe) = 55.93494\,u$ and $m(^{28}_{13}Al) = 27.98191\,u$

**Answer:**

The reaction will be $^{56}_{26}Fe \rightarrow {}^{28}_{13}Al + {}^{28}_{13}Al$

The mass defect of the reaction will be

$$\Delta m = m(^{56}_{26}Fe) - 2m(^{28}_{13}Al)$$

$$\Delta m = 55.93494 - 2 \times 27.98191$$

$$\Delta m = -0.02888\,u$$

Since the mass defect is negative the Q value will also negative and therefore the fission is not energetically possible

**Q. 13.17** The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?

**Answer:**

Number of atoms present in 1 kg(w) of $^{239}_{94}Pu$ =n

$$n = \frac{w \times N_A}{mass\ number\ of\ Pu}$$

$$n = \frac{1000 \times 6.023 \times 10^{23}}{239}$$

$$n = 2.52 \times 10^{24}$$

Energy per fission (E)=180 MeV

Total Energy released if all the atoms in 1 kg $^{239}_{94}Pu$ undergo fission = E × n

$=180 \times 2.52\times10^{24}$

$=4.536\times10^{26}$ MeV

**Q. 13.18** A $1000 MW$ fission reactor consumes half of its fuel in$5.00\ y$. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

**Answer:**

The amount of energy liberated on fission of 1 $^{235}_{92}U$ atom is 200 MeV.

The amount of energy liberated on fission of 1g $^{235}_{92}U$

$$= \frac{200 \times 10^6 \times 1.6 \times 10^{-19} \times 6.023 \times 10^{23}}{235}$$

$$= 8.2 \times 10^{10}\ Jg^{-1}$$

Total Energy produced in the reactor in 5 years

$$= 1000 \times 10^6 \times 0.8 \times 5 \times 365 \times 24 \times 3600$$

$$= 1.261 \times 10^{17}\ J$$

Mass of $^{235}_{92}U$ which underwent fission, m

$$= \frac{1.261 \times 10^{17}}{8.2 \times 10^{10}}$$

=1537.8 kg

The amount present initially in the reactor = 2m

=2×1537.8

=3075.6 kg

**Q. 13.19** How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^2_1H + ^2_1H \rightarrow ^3_2He + n + 3.27\ MeV$$

**Answer:**

The energy liberated on the fusion of two atoms of deuterium= 3.27 MeV

Number of fusion reactions in 2 kg of deuterium = $N_A$×500

The energy liberated by fusion of 2.0 kg of deuterium atoms E

$$= 3.27 \times 10^6 \times 1.6 \times 10^{-19} \times 6.023 \times 10^{23} \times 500$$

$= 1.576 \times 10^{14}\ J$

Power of lamp (P)= 100 W

Time the lamp would glow using E amount of energy is T=

$$= \frac{E}{P}$$

$$= \frac{1.576 \times 10^{14}}{100 \times 3600 \times 24 \times 365}$$

$=4.99 \times 10^4$ years

**Q. 13.20** Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

**Answer:**

For a head-on collision of two deuterons, the closest distances between their centres will be d=2×r

d=2×2.0

d=4.0 fm

$d=4 \times 10^{-15}$ m

charge on each deuteron = charge of one proton=q $=1.6 \times 10^{-19}$ C

The maximum electrostatic potential energy of the system during the head-on collision will be E

$$= \frac{q^2}{4\pi\varepsilon_0 d}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15} \times 1.6 \times 10^{-19}} eV$$

$$= 360\ keV$$

The above basically means to bring two deuterons from infinity to each other would require 360 keV of work to be done or would require 360 keV of energy to be spent.

**Q. 13.21** From the relation $R = R_0 A^{\frac{1}{3}}$, where $R_0$ is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

**Answer:**

Mass of an element with mass number A will be about A u. The density of its nucleus, therefore, would be

$$d = \frac{m}{v}$$

$$d = \frac{A}{\frac{4\pi}{3}(R)^3}$$

$$d = \frac{A}{\frac{4\pi}{3}\left(R_0 A^{\frac{1}{3}}\right)^3}$$

$$d = \frac{3}{4\pi R_0 \hat{}3}$$

As we can see the above density comes out to be independent of mass number A and $R_0$ is constant, so matter density is nearly constant

**Q. 13.22** For the $\beta^+$ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}_Z^AX \rightarrow {}_{Z-1}^{A}Y + v$$

Show that if $\beta^+$emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

**Answer:**

For the electron capture, the reaction would be

$${}_Z^AX \rightarrow {}_{Z-1}^{A}Y + e^+ + v + Q_1$$

The mass defect and q value of the above reaction would be

$$\Delta m_1 = m({}_Z^AX) + m_e - m({}_{Z-1}^{A}Y)$$

$$Q_1 = ([m({}_Z^AX) - m({}_{Z-1}^{A}Y)] + m_e)c^2$$

where $m_N({}_Z^AX)$ and $m_N({}_{Z-1}^{A}Y)$ are the nuclear masses of elements X and Y respectively

For positron emission, the reaction would be

$${}_Z^AX \rightarrow {}_{Z-1}^{A}Y + e^+ + \bar{v} + Q_2$$

The mass defect and q value for the above reaction would be

$$\Delta m_2 = ({}_Z^AX) - m\big((A_{Z-1})Y\big) - m_e$$

$$Q_2 = ([m({}_Z^AX) - m({}_{Z-1}^{A}Y)] - m_e)c^2$$

From the above values, we can see that if $Q_2$ is positive $Q_1$ will also be positive but $Q_1$ being positive does not imply that $Q_2$ will also have to positive.

**Q.13.23** In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}_{12}^{24}Mg(23.98504\ u)$, ${}_{12}^{25}Mg(24.98584\ u)$ and ${}_{12}^{26}Mg(25.98259\ u)$. The natural abundance of is 78.99% by mass. Calculate the abundances of other two isotopes.

**Answer:**

Let the abundances of ${}_{12}^{25}Mg$ and ${}_{12}^{26}Mg$ be x and y respectively.

x+y+78.99=100

y=21.01-x

The average atomic mass of Mg is 24.312 u

$$24.312=\frac{78.99\times 23.98504+x\times 24.98584+(100-x)\times 25.98259}{100}$$

$$x\approx 9.3$$

$$y=21.01-x$$

$$y=21.01-9.3$$

$$y=11.71$$

The abundances of ${}^{25}_{12}Mg$ and ${}^{26}_{12}Mg$ are 9.3% and 11.71% respectively

**Q.13.24 (i)** The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}Ca$ from the following data:

$$m({}^{40}_{20}Ca)=39.962591\,u$$

$$m({}^{41}_{20}Ca)=40.962278\,u$$

$$m({}^{26}_{13}A1)=25.986895\,u$$

$$m({}^{27}_{13}A1)=26.981541\,u$$

**Answer:**

The reaction showing the neutron separation is

$${}^{41}_{20}Ca+E\rightarrow(40_{20})Ca+{}^{1}_{0}n$$

$$E=(m({}^{40}_{20}Ca)+m({}^{1}_{0}n)-m({}^{41}_{20}Ca)c^2$$

$$E=(39.962591+1.008665-40.962278)c^2$$

$$E=(0.008978)u\times c^2$$

But 1u=931.5 MeV/$c^2$

Therefore E=(0.008978)×931.5

E=8.363007 MeV

Therefore to remove a neutron from the ${}^{41}_{20}Ca$ nucleus 8.363007 MeV of energy is required

**Q.13.24 (ii)** The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{27}_{13}A1$ from the following data:

$$m({}^{40}_{20}Ca)=39.962591\,u$$

$$m({}^{41}_{20}Ca)=40.962278\,u$$

$$m({}^{26}_{13}A1)=25.986895\,u$$

$$m(^{27}_{13}A1) = 26.981541\ u$$

**Answer:**

The reaction showing the neutron separation is

$$^{27}_{13}A1 + E \rightarrow ^{26}_{13}A1 + ^{1}_{0}n$$

$$E = \left(m(^{26}_{13}Ca) + m(^{1}_{0}n) - m(^{27}_{13}Ca)\right)c^2$$

$$E = (25.986895 + 1.008665 - 26.981541)c^2$$

$$E = (0.014019)u \times c^2$$

But 1u=931.5 MeV/$c^2$

Therefore E=(0.014019)×931.5

E=13.059 MeV

Therefore to remove a neutron from the $^{27}_{13}A1$ nucleus 13.059 MeV of energy is required

**Q.13.25** A source contains two phosphorous radio nuclides $^{32}_{15}P(T_{\frac{1}{2}} = 14.3d)$ and $^{33}_{15}P(T_{\frac{1}{2}} = 25.3d)$. Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?

**Answer:**

Let initially there be $N_1$ atoms of $^{32}_{15}P$ and $N_2$ atoms of $^{33}_{15}P$ and let their decay constants be $\lambda_1$ and $\lambda_2$ respectively

Since initially the activity of $^{33}_{15}P$ is 1/9 times that of $^{32}_{15}P$ we have

$N_1\lambda_1 = \frac{N_2\lambda_2}{9}$ (i)

Let after time t the activity of $^{33}_{15}P$ be 9 times that of $^{32}_{15}P$

$N_1\lambda_1 e^{-\lambda_1 t} = 9N_2\lambda_2 e^{-\lambda_2 t}$ (ii)

Dividing equation (ii) by (i) and taking the natural log of both sides we get

$$-\lambda_1 t = t = ln81 - \lambda_2 t$$

$$t = \frac{ln81}{\lambda_2 - \lambda_1}$$

where $\lambda_2 = 0.048/day$ and $\lambda_1 = 0.027/day$

t comes out to be 208.5 days

**Q.13.26** Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}Ra \rightarrow ^{209}_{82}Pb + ^{14}_{6}C$$

$$^{223}_{88}Ra \rightarrow ^{219}_{86}Rn + \frac{4}{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

**Answer:**

$$ {}^{223}_{88}Ra \rightarrow {}^{209}_{82}Pb + {}^{14}_{6}C $$

$$ \Delta m = m({}^{223}_{88}Ra) - m({}^{209}_{82}Pb) - m({}^{14}_{6}C) $$

$$ = 223.01850 - 208.98107 - 14.00324 $$

$$ = 0.03419\ u $$

1 u = 931.5 MeV/$c^2$

Q=0.03419×931.5

=31.848 MeV

As the Q value is positive the reaction is energetically allowed

$$ {}^{223}_{88}Ra \rightarrow {}^{219}_{86}Rn + {}^{4}_{2}He $$

$$ \Delta m = m({}^{223}_{88}Ra) - m({}^{219}_{86}Rn) - m({}^{4}_{2}He) $$

$$ = 223.01850 - 219.00948 - 4.00260 $$

$$ = 0.00642\ u $$

1 u = 931.5 MeV/$c^2$

Q=0.00642×931.5

=5.98 MeV

As the Q value is positive the reaction is energetically allowed

**Q.13.27** Consider the fission of ${}^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}Ce$ and ${}^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

$$ m({}^{238}_{92}U) = 238.05079\ u $$

$$ m({}^{140}_{58}Ce) = 139.90543\ u $$

$$ m({}^{99}_{44}Ru) = 98.90594\ u $$

**Answer:**

The fission reaction given in the question can be written as

$$ {}^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{58}Ce + {}^{99}_{44}Ru + 10e^{-} $$

The mass defect for the above reaction would be

$$ \Delta m = m_{N({}^{238}_{92}U)} + m({}^{1}_{0}n) - m_N({}^{140}_{58}Ce) - m_N({}^{99}_{44}Ce) - 10m_e $$

In the above equation, $m_N$ represents nuclear masses

$$ \Delta m = m({}^{238}_{92}U) - 92m_e + m({}^{1}_{0}n) - m({}^{140}_{58}Ce) + 58m_e - m({}^{99}_{44}Ru) + 44m_e - 10m_e $$

$$\Delta m = m({}^{238}_{92}U) + m({}^{1}_{0}n) - m({}^{140}_{58}Ce) - m({}^{99}_{44}Ru)$$

$$\Delta m = 238.05079 + 1.008665 - 139.90543 - 98.90594$$

$$\Delta m = 0.247995u$$

but 1u =931.5 MeV/$c^2$

Q=0.247995×931.5

Q=231.007 MeV

Q value of the fission process is 231.007 MeV

**Q.13.28 (i)** Consider the D–T reaction (deuterium-tritium fusion)

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$$m({}^{2}_{1}H) = 2.014102\ u$$

$m({}^{3}_{1}H) = 3.016049\ u$

**Answer:**

The mass defect of the reaction is

$$\Delta m = m({}^{2}_{1}H) + m({}^{3}_{1}H) - m({}^{4}_{2}He) - m({}^{1}_{0}n)$$

$$\Delta m = 2.014102 + 3.016049 - 4.002603 - 1.008665$$

$$\Delta m = 0.018883u$$

1u = 931.5 MeV/$c^2$

Q=0.018883×931.5=17.59 MeV

**Q.13.28 (b)** Consider the D–T reaction (deuterium–tritium fusion)

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$$

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = $2(3kT/2)$; k = Boltzman's constant, T = absolute temperature.)

**Answer:**

To initiate the reaction both the nuclei would have to come in contact with each other.

Just before the reaction the distance between their centres would be 4.0 fm.

The electrostatic potential energy of the system at that point would be

$$U = \frac{q^2}{4\pi\varepsilon_0 d}$$

$$U = \frac{9 \times 10^9 (1.6 \times 10^{-19})^2}{4 \times 10^{-15}}$$

$$U = 5.76 \times 10^{-14} J$$

The same amount of Kinetic Energy K would be required to overcome the electrostatic forces of repulsion to initiate the reaction

It is given that $K = 2 \times \frac{3kT}{2}$

Therefore the temperature required to initiate the reaction is

$$T = \frac{K}{3k}$$

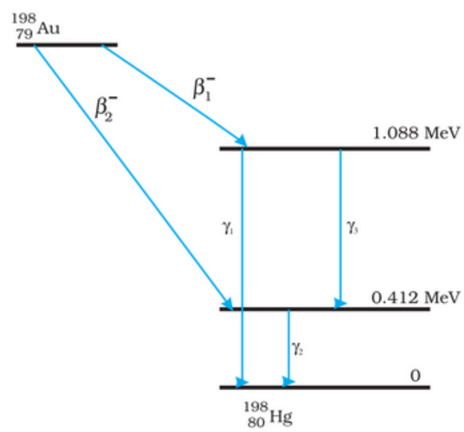
$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}}$$

$$= 1.39 \times 10^9 K$$

**Q. 13.29** Obtain the maximum kinetic energy of β- particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

$$m(^{198}Au) = 197.968233\ u$$

$$m(^{198}Hg) = 197.966760\ u$$




**Answer:**

$\gamma_1$ decays from 1.088 MeV to 0 V

Frequency of $\gamma_1$ is

$$v_1 = \frac{1.088 \times 10^6 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}}$$

$$v_1 = 2.367 \times 10^{20} Hz$$

Plank's constant, h=6.62×$10^{-34}$ Js $E = hv$

Similarly, we can calculate frequencies of $\gamma_2$ and $\gamma_3$

$$v_2 = 9.988 \times 10^{19}\ Hz$$

$$v_3 = 1.639 \times 10^{20}\ Hz$$

The energy of the highest level would be equal to the energy released after the decay

Mass defect is

$$\Delta m = m({}^{196}_{79}U) - m({}^{196}_{80}Hg)$$

$$\Delta m == 197.968233 - 197.966760$$

$$\Delta m = 0.001473u$$

We know 1u = 931.5 MeV/$c^2$

Q value= 0.001473×931.5=1.3721 MeV

The maximum Kinetic energy of $\beta_1^-$ would be 1.3721-1.088=0.2841 MeV

The maximum Kinetic energy of $B_2^-$ would be 1.3721-0.412=0.9601 MeV

**Q. 13.30** Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of 235U in a fission reactor.

**Answer:**

(a) ${}^1_1H\,{}^1_1H + {}^1_1H + {}^1_1H + {}^1_1H \rightarrow {}^4_2He$

The above fusion reaction releases the energy of 26 MeV

Number of Hydrogen atoms in 1.0 kg of Hydrogen is 1000$N_A$

Therefore 250$N_A$ such reactions would take place

The energy released in the whole process is $E_1$

$$= 250 \times 6.023 \times 10^{23} \times 26 \times 10^6 \times 1.6 \times 10^{-19}$$

(b) The energy released in fission of one ${}^{235}_{92}U$ atom is 200 MeV

Number of ${}^{235}_{92}U$ atoms present in 1 kg of ${}^{235}_{92}U$ is N

$$N = \frac{1000 \times 6.023 \times 10^{23}}{235}$$

$$N = 2.562 \times 10^{24}$$

The energy released on fission of N atoms is $E_2$

$$E = 2.562 \times 10^{24} \times 200 \times 10^6 \times 1.6 \times 10^{-19}$$

$$E = 8.198 \times 10^{13}\ J$$

$$\frac{E_1}{E_2} = \frac{6.2639 \times 10^{14}}{8.198 \times 10^{13}} \approx 8$$

**Q. 13.31** Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of 235U to be about 200MeV.

**Answer:**

Let the amount of energy to be produced using nuclear power per year in 2020 is E

$$E = \frac{200000 \times 10^6 \times 0.1 \times 365 \times 24 \times 3600}{0.25} J$$

(Only 10% of the required electrical energy is to be produced by Nuclear power and only 25% of thermo-nuclear is successfully converted into electrical energy)

Amount of Uranium required to produce this much energy is M

($N_A=6.023\times10^{23}$, Atomic mass of Uranium is 235 g)

$=3.076\times10^4$ kg