The sum of the two irrational numbers...

యార్లగడ్డ వనంరాజు
సబ్జెక్టు నిపుణులు

REAL NUMBERS

Euclid's Division Lemma

Let 'a' and 'b' any two positive integers. Then there exist two unique whole numbers q, r such that

a = bq + r, o ≤ r ≤ b

Here 'a' is called the dividend, b is called the divisor, q is called the quotient, r is called the remainder. Alternatively

Dividend = Divisor × Quotient + Remainder

1. Use Euclid's division algorithm to find the HCF of 135, 225.

Sol: Given numbers are 135, 225.

Start with the larger integer, that is 225.

Apply the division lemma to 225, 135,

225 = 135 × 1 + 90

Since the remainder 90 ≠ 0, we apply the division lemma to 135 and 90, to get 135 = 90 × 1 + 45

We consider the new divisor 90, the new remainder 45, apply the division lemma to get 90 = 45 × 2 + 0

The remainder has now become zero, so our procedure stops.

Since the divisor at this stage is 45.

∴ The HCF of 225, 135 is 45.

2. Use Euclid's division lemma to show that the square of any positive integer is of the form 3m or 3m+1 for some integer m.

Sol: Let 'a' be any positive integer. We apply the division lemma with a, b= 3

Since o ≤ r < 3, the possible remainders are 0, 1, 2. i.e. 'a' can be 3p or 3p+1 or 3p+2, where 'p' is the quotient. Now$(3p)^2 = 9p^2 = 3(3p^2)$

Which can be written in the form 3m

Again $(3p+1)^2 = 9p^2 + 6p+1 = 3(3p^2+2p)+1$

Which can be written in the form 3m+1.

Lastly $(3p+2)^2 = 9p^2 + 12p + 4$

$= (9p^2 + 12p + 3) + 1$

$= 3(3p^2 + 4p + 1) + 1$

Which can be written in the form 3m+1.

∴ The square of any positive integer is either of the form 3m or 3m+1 for some integer m.

The Fundamental Theorem of Arithmetic

Every composite number can be expressed (Factored) as a product of primes, this factorization is unique, apart from the order in which the prime factors occur.

In general, given a composite number *x*, we factorize it as

$x = p_1p_2...p_n$, where $p_1, p_2, ... p_n$ are primes and written in ascending order. i.e. $p_1 \le p_2 \le ... \le p_n$

Ex: 156 = 2 × 2 × 3 × 13

$= 2^2 \times 3^1 \times 13^1$

3825 = 3 × 3 × 5 × 5 × 17

$= 3^2 \times 5^2 \times 17$

1. Express each of the following number as a product of its prime factors.

i) 5005 ii) 7429

(Communication)

Sol:

i) 5005 = 5 × 7 × 11 × 13

ii) 7429 = 17 × 19 × 23

2. Check whether 6^n can end with the digit '0' for any natural numbers n. *(Reasoning proof)*

Sol: n is any natural number.

If n = 1, then $6^n = 6^1 = 6$

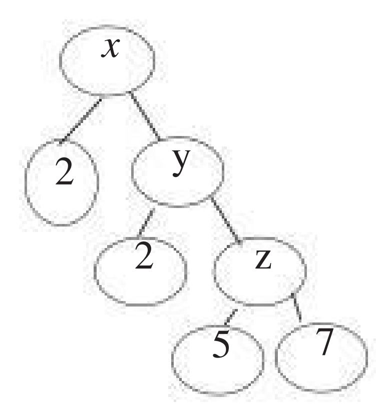
If n = 2, then $6^n = 6^2 = 36$

If n = 3, then $6^n = 6^3 = 216$ ....

From the above, we can say that 6^n can be end with 6 only.

∴ For any natural number n, 6n cannot end with the digit '0'.

3. Find the value of x, y, z in the given factor tree



(Visualization & Representation)

Sol: From the factor tree.

z = 5 × 7 = 35

y = 2 × z = 2 × 35 = 70

x = 2 × y = 2 × 70 = 140.

LCM

LCM: The LCM of two positive integers is defined as the product of the greatest power of each prime factor, involved in the numbers.

Ex: LCM of 336, 54.

$336 = 2^4 \times 3^1 \times 7^1$

$54 = 2 \times 3^3$

LCM of 336, 54 = product of the greatest power of each prime factors of the numbers $= 2^4 \times 3^3 \times 7^1 = 3024$

HCF

The HCF of two positive integers is defined as the product of the smallest power of each common prime factor in the numbers.

Ex: HCF of 336, 54

$336 = 2^4 \times 3^1 \times 7^1$, $54 = 2 \times 3^3$

HCF of 336, 54 = product of the smallest power of each common prime factors of the numbers

$2^1 \times 3^1 = 6$

- If a, b are any two positive integers, then HCF (a, b) ×LCM (a, b) = a×b.

1. Find the HCF, LCM of the following integers by the prime factorization method.

i) 12, 15, 21

ii) 72, 108 *(Problem Solving)*

Sol: 1) $12 = 2^2 \times 3$, $15 = 3 \times 5$,

21 = 3×7

$LCM = 2^2 \times 3 \times 5 \times 7 = 420$

HCF = 3

ii) $72 = 2^3 \times 3^2$, $108 = 2^2 \times 3^3$

$LCM = 2^3 \times 3^3 = 216$,

$HCF = 2^2 \times 3^2 = 36$

2. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together? *(connection)*

Sol: Required number of minutes is the LCM of 9, 12, 15

$= 2^2 \times 3^2 \times 5 = 180$ minutes = 3 hours

$9 = 3^2$

$12 = 2^2 \times 3$

$15 = 3 \times 5$

∴ Three bells will toll together again, after 3 hours

Terminating or Non Terminating, Repeating (recurring) Decimals in Rational Numbers

Let $x = \frac{p}{q}$ be a rational number.If the prime factorization of q, is in the form of $2^n.5^m$, where n, m are non-negative integers, then

$x = \frac{p}{q}$ is a terminating decimal.

if q is not in the form of $2^n.5^m$, then x is a non terminating, repeating (recurring) decimal. Converse also true.

Ex: i) $\frac{16}{125} = \frac{16}{5^3}$

Here $q = 5^3 = 2^0 \times 5^3$, which is in the form $2^n.5^m$ (n=0, m=3), so

the rational number $\frac{16}{125}$ is a terminating decimal.

ii) $\frac{41}{75} = \frac{41}{3\ 5^2}$

Here $q = 3^1 \times 5^2$, which is not in the form $2^n.5^m$

$\therefore \frac{41}{75}$ is a non terminating, repeating decimal.

Rational numbers (Q): A number which can be written either in the form of terminating decimal or non-terminating repeating decimal is called a rational number.

Set of Rational no.s

$$Q = \left\{\frac{p}{q} / p, q \in z, q \ne 0, p, q \text{ are co-primes}\right\}$$

Irrational numbers (Q'): A number which cannot be written in the form of non-terminatring non-repeating is called irrational number.

Ex: $\sqrt{2} = 1.414...$, $\sqrt{3} = 1.732...$, $\pi = 3.14...$ etc.

- An irrational number between a, b is $\sqrt{ab}$ where a, b are not perfect square numbers
- $\sqrt{P}$ is irrational, where p is prime
  Ex: $\sqrt{2}, \sqrt{3}, \sqrt{5}$, ...
- Sum (or difference) of a rational, an irrational number is irrational.
- The product (or quotient) of a non-zero rational, irrational number is irrational.
- The sum (or product) of the two irrational numbers need not be irrational.
- We will prove the irrationality of numbers by using the 'method of contradiction'

1. Write the decimal expansion of the following rational numbers without actual division.

(Communication)

i) $\frac{13}{3125}$ ii) $\frac{7218}{3^2 - 5^2}$

Sol:

i) $\frac{13}{3125} = \frac{13}{5^5} = \frac{13\ \ 2^5}{5^5\ \ 2^5}$

$\frac{13\ \ 32}{(5\ \ 2)^5} = \frac{416}{10^5} = 0.00416$

ii) $\frac{7218}{3^2.5^2} = \frac{9\ \ 802}{9\ \ 5^2} = \frac{802}{5^2}$

$= \frac{802\ \ 2^2}{5^2\ \ 2^2} = \frac{802\ \ 4}{(5\ \ 2)^2}$

$= \frac{3208}{10^2} = 32.08$

2. Without performing division, state whether the following rational numbers will have a terminating decimal form or a non-terminating, repeating decimal form. *(Reasoning proof)*

i) $\frac{15}{1600}$ ii) $\frac{64}{455}$

Sol:

i) $\frac{15}{1600} = \frac{5\ \ 3}{5\ \ 320} = \frac{3}{320} = \frac{3}{2^6\ \ 5}$

$(\because 320 = 2^6 \times 5)$

Here $q = 2^6 \times 5$, which is in the form $2^n \times 5^m$ (n=6, m = 1)

$\therefore \frac{15}{1600}$ is a terminating decimal

ii) $\frac{64}{455} = \frac{64}{5\ \ 7\ \ 13}$

Here q = 5×7×13, is which is not in the form $2^n.5^m$

$\therefore \frac{64}{455}$ is a non terminating, repeating decimal.

Model Questions

1 Mark Questions

1. Express 156 as a product of its Prime factors.

Sol: 156 = 2 78 = 2 2 39

$= 2\ \ 2\ \ 3\ \ 13 = 2^2\ \ 3\ \ 13$

2. Explain why 3 5 7+7 is a composite number.

Sol: $3\ \ 5\ \ 7+7 = (3\ \ 5\ \ 1+1)\ \ 7 = 16\ \ 7 = 2^4\ \ 7$

Given number can be expressed as a product of primes.

∴ By fundamental theorem of Arithmetic it is a composite number.

3. "The sum of the two irrational numbers need not be irrational. Give example.

Sol: If $a=\sqrt{3}$, $b = -\sqrt{3}$ are irrational. but a+b =0, which is

4. What is the last digit of 6^{2019}

Sol: $6^1 = 6$

$6^2 = 6\ \ 6 = 36$

$6^3 = 6\ \ 6\ \ 6 = 216$

for any positive integer n, 6^n can end with 6.

∴The last digit of 6^{2019} is 6.