JEE Main 2019

Forenoon Session – Slot 1

January 9 – Paper 1

JEE Main 2019 Paper 1 Question Paper and Answer Key: January 9, Shift 1
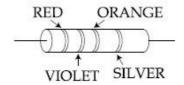
Section : Physics

Q.1 A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is :

Options 1. 1.1 cm away from the lens

2. 0

3. 0.55 cm towards the lens

4. 0.55 cm away from the lens

Q.2 A resistance is shown in the figure. Its value and tolerance are given respectively by :

RED   ORANGE

VIOLET   SILVER

Options 1. 270 Ω, 10 %
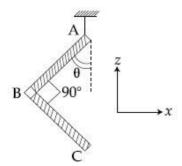
2. 27 kΩ, 10 %

3. 27 kΩ, 20 %

4. 270 Ω, 5 %

Q.3

Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm², is v. If the electron density in copper is $9 \times 10^{28}/m^3$ the value of v in mm/s is close to (Take charge of electron to be $= 1.6 \times 10^{-19}$ C)

Options
1. 0.02

2. 3

3. 2

4. 0.2

Q.4

An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB=BC, and the angle made by AB with downward vertical is θ, then :

Options

1. $\tan\theta = \dfrac{1}{2\sqrt{3}}$

2. $\tan\theta = \dfrac{1}{2}$

3. $\tan\theta = \dfrac{2}{\sqrt{3}}$

4. $\tan\theta = \dfrac{1}{3}$

Q.5

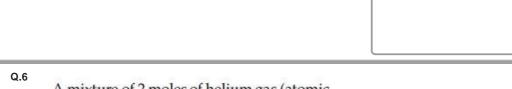A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is :

1. $y = x^2 + \text{constant}$

2. $y^2 = x + \text{constant}$

3. $y^2 = x^2 + \text{constant}$

4. $xy = \text{constant}$

Q.6

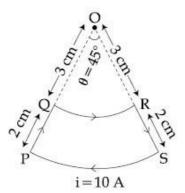A mixture of 2 moles of helium gas (atomic mass = 4 u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\dfrac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})} \right]$, is close to :
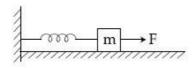
1. 3.16

2. 0.32

3. 0.45

4. 2.24

Q.7 A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to :

$i = 10$ A

1. 1.0×10^{-7} T

3

2. 1.5×10^{-7} T

3. 1.5×10^{-5} T

4. 1.0×10^{-5} T

Q.8 A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is :

Options

1. $\dfrac{2F}{\sqrt{mk}}$

2. $\dfrac{F}{\pi\sqrt{mk}}$

3. $\dfrac{\pi F}{\sqrt{mk}}$

4. $\dfrac{F}{\sqrt{mk}}$

Q.9 For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is :
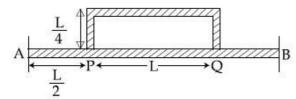
Options

1. $\dfrac{R}{\sqrt{5}}$

2. $\dfrac{R}{\sqrt{2}}$

3. R

4. $R\sqrt{2}$

Q.10 Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio :

Options 1. $16:9$

2. $25:9$

3. $4:1$

4. $5:3$

Q.11 Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to :

(Energy of photon $= \dfrac{1240}{\lambda(\text{in nm})} eV$)

Options 1. 1.8

2. 2.5

3. 5.6

4. 1.4

Q.12

Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section 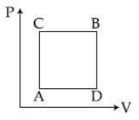as AB and length $\dfrac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to :

1. 45 °C

2. 75 °C

3. 60 °C

4. 35 °C

Q.13 A gas can be taken from A to B via two different processes ACB and ADB.
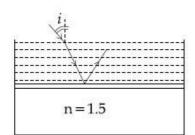

When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is :

Options 1. 40 J

2. 80 J

3. 100 J

4. 20 J

Q.14 Consider a tank made of glass(refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is :

n = 1.5

Options

1. $\sqrt{\dfrac{5}{3}}$

2. $\dfrac{3}{\sqrt{5}}$

3. $\dfrac{5}{\sqrt{3}}$

4. $\dfrac{4}{3}$

Q.15 Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 m^2/(V.s) then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to :

Options 1. $2\,\Omega m$

2. $4\,\Omega m$

3. $0.4\,\Omega m$

4. $0.2\,\Omega m$

Q.16 A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be:

Options
1. $18.9 \times 10^{-8}\ \hat{k}$ T

2. $2.1 \times 10^{-8}\ \hat{k}$ T

3. $6.3 \times 10^{-8}\ \hat{k}$ T

4. $18.9 \times 10^{8}\ \hat{k}$ T

Q.17 Three charges $+Q$, q, $+Q$ are placed respectively, at distance, 0, $d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x=0$, is zero, then value of q is:

Options
1. $-Q/4$

2. $+Q/2$

3. $+Q/4$

4. $-Q/2$

Q.18 A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is:
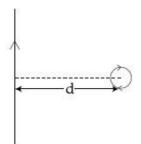
Options
1. 2.0 %

2. 2.5 %

3. 1.0 %

4. 0.5 %

Q.19 A sample of radioactive material A, that has an activity of 10 mCi($1 \ Ci = 3.7 \times 10^{10}$ decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively :

Options 1. 5 days and 10 days

2. 10 days and 40 days

3. 20 days and 5 days

4. 20 days and 10 days

Q.20 A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is $60 \ ms^{-1}$. When the car has acceleration a, the wave-speed increases to $60.5 \ ms^{-1}$. The value of a, in terms of gravitational acceleration g, is closest to :

Options

1. $\dfrac{g}{30}$

2. $\dfrac{g}{5}$

3. $\dfrac{g}{10}$

4. $\dfrac{g}{20}$

Q.21

A conducting circular loop made of a thin wire, has area $3.5 \times 10^{-3} \, m^2$ and resistance $10 \, \Omega$. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to :

1. 14 mC

2. 7 mC

3. 21 mC

4. 6 mC

Q.22 An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d>>a). If the loop applies a force F on the wire then :
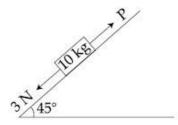

Options 1. $F = 0$

2. $F \propto \left(\dfrac{a}{d} \right)$

3. $F \propto \left(\dfrac{a^2}{d^3} \right)$

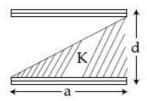4. $F \propto \left(\dfrac{a}{d} \right)^2$

Q.23

A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward ?
(take $g = 10$ ms^{-2})

Options 1. 32 N

2. 18 N

3. 23 N

4. 25 N

Q.24 A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is :

Options

1. $\dfrac{K\epsilon_0 a^2}{2d(K+1)}$

2. $\dfrac{K\epsilon_0 a^2}{d(K-1)}\ln K$

3. $\dfrac{K\epsilon_0 a^2}{d}\ln K$

4. $\dfrac{1}{2}\dfrac{K\epsilon_0 a^2}{d}$

…

Q.25 A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion $\alpha/°C$. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is :

Options

1. $\dfrac{F}{A\alpha\Delta T}$

2. $\dfrac{F}{A\alpha(\Delta T - 273)}$

3. $\dfrac{F}{2A\alpha\Delta T}$

4. $\dfrac{2F}{A\alpha\Delta T}$

Q.26 A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is :

Options 1. 285 A/m

2. 2600 A/m

3. 520 A/m

4. 1200 A/m

Q.27

Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C,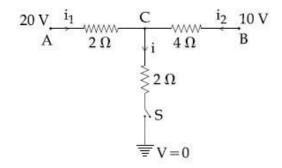 also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m ?

```
   A     B     C
  | m | | m |  | M |
_____
```

Options 1. 5

2. 2

3. 4

4. 3

Q.28 When the switch S, in the circuit shown, is closed, then the value of current i will be :

```
20 V  i1        C         i2  10 V
  A——www——         ——www——→  B
        2Ω    |i      4Ω
              |
             ≷ 2Ω
              |
              \ S
              |
            ═ V=0
```

Options 1. 3 A

2. 5 A

3. 4 A

4. 2 A

Q.29

If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is :
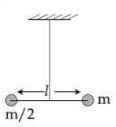
Options

1. $\dfrac{L}{m}$

2. $\dfrac{4L}{m}$

3. $\dfrac{L}{2m}$

4. $\dfrac{2L}{m}$

Q.30

Two masses m and $\dfrac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be :

m/2

Options

1. $\dfrac{3k\theta_0{}^2}{l}$

2. $\dfrac{2k\theta_0{}^2}{l}$

3. $\dfrac{k\theta_0{}^2}{l}$

4. $\dfrac{k\theta_0{}^2}{2l}$

Section: Chemistry

Q.1 Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is :

Options

1. Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively.

2. both are paramagnetic with three unpaired electrons.

3. both absorb energies corresponding to their complementary colors.

4. Δ_0 value for (A) is less than that of (B).

Q.2 The correct decreasing order for acid strength is :

Options

1. $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

2. $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

3. $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

4. $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

Q.3 The major product of following reaction is :

$$R-C\equiv N \xrightarrow[\text{(2) } H_2O]{\text{(1) AlH(i-Bu)}_2} ?$$

Options 1. RCOOH

2. $RCONH_2$

3. RCHO

4. RCH_2NH_2

The highest value of the calculated spin-only magnetic moment (in BM) among all the transition metal complexes is :

1. 5.92

2. 6.93

3. 3.87

4. 4.90

0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume $10\,m^3$ at $1000\,K$. Given R is the gas constant in $JK^{-1}mol^{-1}$, x is :

1. $\dfrac{2R}{4+R}$

2. $\dfrac{2R}{4-R}$

3. $\dfrac{4+R}{2R}$

4. $\dfrac{4-R}{2R}$

Q.6 The one that is extensively used as a piezoelectric material is :

1. tridymite

2. amorphous silica

3. quartz

Options

4. mica

Q.7 Correct statements among a to d regarding silicones are :

(a) They are polymers with hydrophobic character.

(b) They are biocompatible.

(c) In general, they have high thermal stability and low dielectric strength.

(d) Usually, they are resistant to oxidation and used as greases.

Options 1. (a), (b), (c) and (d)

2. (a), (b) and (c) only

3. (a) and (b) only

4. (a), (b) and (d) only

Q.8 The major product of the following reaction is :

(i) Br_2
(ii) EtOH

Options

1.

2.

3.

4.

Q.9 In general, the properties that decrease and increase down a group in the periodic table, respectively, are :

Options 1. atomic radius and electronegativity.

2. electron gain enthalpy and electronegativity.

3. electronegativity and atomic radius.

4. electronegativity and electron gain enthalpy.

Q.10 A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in $mol\ kg^{-1}$ is :

Options 1. 12

2. 4

3. 8

4. 16

Q.11

The correct match between Item-I and Item-II is :

	Item-I (drug)		Item-II (test)
A	Chloroxylenol	P	Carbylamine test
B	Norethindrone	Q	Sodium hydrogen-carbonate test
C	Sulphapyridine	R	Ferric chloride test
D	Penicillin	S	Bayer's test

Options 1. $A \rightarrow R$; $B \rightarrow P$; $C \rightarrow S$; $D \rightarrow Q$

2. $A \rightarrow Q$; $B \rightarrow S$; $C \rightarrow P$; $D \rightarrow R$

3. $A \rightarrow R$; $B \rightarrow S$; $C \rightarrow P$; $D \rightarrow Q$

4. $A \rightarrow Q$; $B \rightarrow P$; $C \rightarrow S$; $D \rightarrow R$

Q.12 A water sample has ppm level concentration of the following metals : $Fe = 0.2$; $Mn = 5.0$; $Cu = 3.0$; $Zn = 5.0$. The metal that makes the water sample unsuitable for drinking is :

Options 1. Cu

2. Mn

3. Fe

4. Zn

Q.13 The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4 = 303$ g mol^{-1})

Options 1. 22.8

2. 15.2

3. 7.6

4. 11.4

Q.14 Which one of the following statements regarding Henry's law is not correct ?

Options
1. Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids.

2. Different gases have different K_H (Henry's law constant) values at the same temperature.

3. The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

4. The value of K_H increases with increase of temperature and K_H is function of the nature of the gas

Q.15 The following results were obtained during kinetic studies of the reaction ;

$2A + B \rightarrow Products$

Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is :

Options
1. 5

2. 10

3. 1

4. 100

Q.16 Major product of the following reaction is :

Options

1.

2.

3.

4.

Q.17 The alkaline earth metal nitrate that does not crystallise with water molecules, is :

Options
1. $Mg(NO_3)_2$

2. $Sr(NO_3)_2$

3. $Ca(NO_3)_2$

4. $Ba(NO_3)_2$

Q.18

20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is : [pk_b of $NH_4OH = 4.7$].

Options 1. 5.2

2. 9.0

3. 5.0

4. 9.4

Q.19 Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. $\dfrac{x}{m}$ is proportional to :

$Log\ \dfrac{x}{m}$ 2 unit

4 unit

Log P

Options 1. p^2

2. $p^{1/4}$

3. $p^{1/2}$

4. p

Q.20 Which amongst the following is the strongest acid ?

Options 1. $CHBr_3$

2. CHI_3

3. $CH(CN)_3$

4. $CHCl_3$

Q.21 The ore that contains both iron and copper is :

Options 1. copper pyrites

2. malachite

3. dolomite

4. azurite

Q.22 For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n$, the plot of wave number $(\bar{v})$ against $\left(\dfrac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit)

Options 1. Linear with intercept $-R_H$

2. Non linear

3. Linear with slope R_H

4. Linear with slope $-R_H$

Q.23 The isotopes of hydrogen are :

Options 1. Tritium and protium only

2. Protium and deuterium only

3. Protium, deuterium and tritium

4. Deuterium and tritium only

Q.24 According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ?

Options

1. Li_2^+ is unstable and Li_2^- is stable

2. Li_2^+ is stable and Li_2^- is unstable

3. Both are stable

4. Both are unstable

Q.25 The major product of the following reaction is :

(1) KOH (aqueous)

(2) CrO_3/H^+

(3) H_2SO_4/Δ

Options

1.

2.

3.

4.

Q.26 Aluminium is usually found in $+3$ oxidation state. In contrast, thallium exists in $+1$ and $+3$ oxidation states. This is due to :

1. inert pair effect

2. diagonal relationship

3. lattice effect

4. lanthanoid contraction

Q.27 The increasing order of pKa of the following amino acids in aqueous solution is :
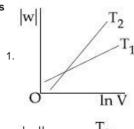
Gly Asp Lys Arg

1. Asp < Gly < Arg < Lys

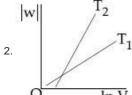2. Gly < Asp < Arg < Lys

3. Asp < Gly < Lys < Arg
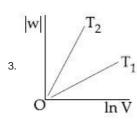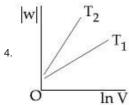
4. Arg < Lys < Gly < Asp

Q.28 Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 $(T_1 < T_2)$. The correct graphical depiction of the dependence of work done (w) on the final volume (V) is :
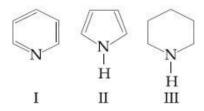
1.

2.

3.

4.

Q.29 Arrange the following amines in the decreasing order of basicity :

I   II   III

Options 1. I > II > III

2. III > I > II

3. III > II > I

4. I > III > II

Q.30 The compounds A and B in the following reaction are, respectively :

$$\boxed{\bigcirc} \xrightarrow{HCHO + HCl} A \xrightarrow{AgCN} B$$

Options

1. A = Benzyl alcohol, B = Benzyl cyanide

2. A = Benzyl chloride, B = Benzyl cyanide

3. A = Benzyl alcohol, B = Benzyl isocyanide

4. A = Benzyl chloride, B = Benzyl isocyanide

Section: Mathematics

Q.1

The value of $\int\limits_0^\pi \left|\cos x\right|^3 dx$ is :

Options 1. 0

2. $\dfrac{4}{3}$

3. $\dfrac{2}{3}$

4. $-\dfrac{4}{3}$

Q.2 The maximum volume (in cu.m) of the right circular cone having slant height 3 m is :

Options 1. 6π

2. $3\sqrt{3}\ \pi$

3. $\dfrac{4}{3}\pi$

4. $2\sqrt{3}\ \pi$

Q.3

For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x \sqrt{\frac{2\sin(x^2 - 1) - \sin 2(x^2 - 1)}{2\sin(x^2 - 1) + \sin 2(x^2 - 1)}}\, dx \text{ is}$$

equal to :

(where c is a constant of integration)

1. $\log_e \left| \dfrac{1}{2} \sec^2 (x^2 - 1) \right| + c$

2. $\dfrac{1}{2} \log_e |\sec(x^2 - 1)| + c$

3. $\dfrac{1}{2}\log_e \left| \sec^2 \left(\dfrac{x^2 - 1}{2} \right) \right| + c$

4. $\log_e \left| \sec \left(\dfrac{x^2 - 1}{2} \right) \right| + c$

Q.4 If $y = y(x)$ is the solution of the differential

equation, $x\dfrac{dy}{dx} + 2y = x^2$ satisfying

$y(1) = 1$, then $y\left(\dfrac{1}{2}\right)$ is equal to :

1. $\dfrac{7}{64}$

2. $\dfrac{1}{4}$

3. $\dfrac{49}{16}$

4. $\dfrac{13}{16}$

Q.5

Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

Options
1. $(5, 2\sqrt{6})$

2. $(8, 6)$

3. $(6, 4\sqrt{2})$

4. $(4, -4)$

Q.6 Let $0 < \theta < \dfrac{\pi}{2}$. If the eccentricity of the

hyperbola $\dfrac{x^2}{\cos^2\theta} - \dfrac{y^2}{\sin^2\theta} = 1$ is greater

than 2, then the length of its latus rectum lies in the interval :

Options 1. $(3, \infty)$

2. $(3/2, 2]$

3. $(2, 3]$

4. $(1, 3/2]$

Q.7 For $x \in R - \{0, 1\}$, let $f_1(x) = \dfrac{1}{x}, f_2(x) = 1 - x$

and $f_3(x) = \dfrac{1}{1-x}$ be three given

functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to :

Options 1. $f_3(x)$

2. $\dfrac{1}{x} f_3(x)$

3. $f_2(x)$

4. $f_1(x)$

Q.8 Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$

be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$

and $\vec{a} \cdot \vec{c} = 4$, then $\left| \vec{c} \right|^2$ is equal to :

Options
1. $\dfrac{19}{2}$

2. 9

3. 8

4. $\dfrac{17}{2}$

Q.9 If a, b and c be three distinct real numbers
in G.P. and $a + b + c = xb$, then x cannot be :

Options
1. -2

2. -3

3. 4

4. 2

Q.10 If $\cos^{-1}\left(\dfrac{2}{3x}\right) + \cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2}$ $\left(x > \dfrac{3}{4}\right)$,

then x is equal to :

Options
1. $\dfrac{\sqrt{145}}{12}$

2. $\dfrac{\sqrt{145}}{10}$

3. $\dfrac{\sqrt{146}}{12}$

4. $\dfrac{\sqrt{145}}{11}$

Q.11 Equation of a common tangent to the circle,
$x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$,
is :

Options 1. $2\sqrt{3}y = 12x + 1$

2. $\sqrt{3}y = x + 3$

3. $2\sqrt{3}y = -x - 12$

4. $\sqrt{3}y = 3x + 1$

Q.12 The system of linear equations
$$x + y + z = 2$$
$$2x + 3y + 2z = 5$$
$$2x + 3y + (a^2 - 1)z = a + 1$$

Options 1. is inconsistent when $a = 4$

2. has a unique solution for $|a| = \sqrt{3}$

3. has infinitely many solutions for $a = 4$

4. is inconsistent when $|a| = \sqrt{3}$

Q.13
If the fractional part of the number $\dfrac{2^{403}}{15}$ is

$\dfrac{k}{15}$, then k is equal to :

Options

1. 6

2. 8

3. 4

4. 14

Q.14 The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x+2y-z-5=0$ and intersecting the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} \text{ is :}$$

Options

1. $\dfrac{x-4}{2} = \dfrac{y+3}{1} = \dfrac{z+1}{4}$

2. $\dfrac{x+4}{1} = \dfrac{y-3}{1} = \dfrac{z-1}{3}$

3. $\dfrac{x+4}{3} = \dfrac{y-3}{-1} = \dfrac{z-1}{1}$

4. $\dfrac{x+4}{-1} = \dfrac{y-3}{1} = \dfrac{z-1}{1}$

Q.15 Consider the set of all lines $px+qy+r=0$ such that $3p+2q+4r=0$. Which one of the following statements is **true** ?

Options   The lines are concurrent at the point

1. $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$.

2. Each line passes through the origin.

3. The lines are all parallel.

4. The lines are not concurrent.

Q.16

$$\lim_{y \to 0} \frac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$$

Options

1. exists and equals $\dfrac{1}{4\sqrt{2}}$

2. exists and equals $\dfrac{1}{2\sqrt{2}\,(\sqrt{2} + 1)}$

3. exists and equals $\dfrac{1}{2\sqrt{2}}$

4. does not exist

Q.17 The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y - axis also passes through the point :

Options
1. $(-3, 0, -1)$

2. $(-3, 1, 1)$

3. $(3, 3, -1)$

4. $(3, 2, 1)$

Q.18 If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan \theta|$ is equal to :

Options
1. $\dfrac{4}{9}$

2. $\dfrac{8}{15}$

3. $\dfrac{7}{17}$

4. $\dfrac{8}{17}$

Q.19

If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix

A^{-50} when $\theta = \dfrac{\pi}{12}$, is equal to :

Options

1. $\begin{bmatrix} \dfrac{1}{2} & -\dfrac{\sqrt{3}}{2} \\ \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \end{bmatrix}$

2. $\begin{bmatrix} \dfrac{\sqrt{3}}{2} & -\dfrac{1}{2} \\ \dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \end{bmatrix}$

3. $\begin{bmatrix} \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \\ -\dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \end{bmatrix}$

4. $\begin{bmatrix} \dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \\ -\dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \end{bmatrix}$

Q.20

If the Boolean expression

$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to

$p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the

ordered pair $(\oplus, \odot)$ is :

Options 1. $(\vee, \wedge)$

2. $(\vee, \vee)$

3. $(\wedge, \vee)$

4. $(\wedge, \wedge)$

Q.21 5 students of a class have an average height 150 cm and variance 18 cm². A new student, whose height is 156 cm, joined them. The variance (in cm²) of the height of these six students is :

1. 16

2. 22

3. 20

4. 18

Q.22 For any $\theta \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$, the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

1. $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

2. $13 - 4\cos^6\theta$

3. $13 - 4\cos^2\theta + 6\cos^4\theta$

4. $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

Q.23 The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

1. $\dfrac{8}{3}$

2. $\dfrac{32}{3}$

3. $\dfrac{56}{3}$

4. $\dfrac{14}{3}$

Let $a_1, a_2, \ldots, a_{30}$ be an A.P., $S = \sum\limits_{i=1}^{30} a_i$ and

$T = \sum\limits_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$,

then a_{10} is equal to :

Options 1. 52

2. 57

3. 47

4. 42

Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if} & x \le 1 \\ a + bx, & \text{if} & 1 < x < 3 \\ b + 5x, & \text{if} & 3 \le x < 5 \\ 30, & \text{if} & x \ge 5 \end{cases}$$

Then, f is :

Options 1. continuous if $a = 5$ and $b = 5$

2. continuous if $a = -5$ and $b = 10$

3. continuous if $a = 0$ and $b = 5$

4. not continuous for any values of a and b

Let $A = \left\{ \theta \in \left(-\dfrac{\pi}{2}, \pi \right) : \dfrac{3 + 2i \sin\theta}{1 - 2i \sin\theta} \right.$ is

purely imaginary $\left. \vphantom{\dfrac{3}{1}} \right\}$. Then the sum of the

elements in A is :

1. $\dfrac{5\pi}{6}$

2. π

3. $\dfrac{3\pi}{4}$

4. $\dfrac{2\pi}{3}$

Q.27 Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is :

1. 500

2. 200

3. 300

4. 350

Q.28 Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

1. -256

2. 512

3. -512

4. 256

Q.29 Three circles of radii a, b, c (a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

Options

1. $\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$

2. $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{c}}$

3. a, b, c are in A.P.

4. $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

Q.30 Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X=1) + P(X=2)$ equals :

Options 1. 49/169

2. 52/169

3. 24/169

4. 25/169