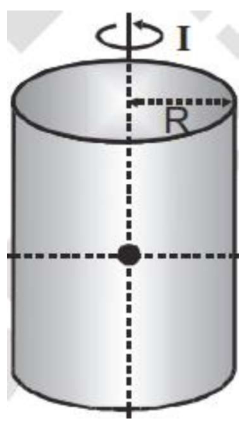
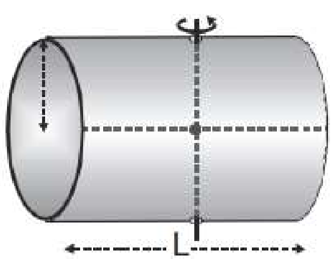
**WE-34:** ***A uniform cylinder has radius R and length L. If the moment of inertia of this cylinder about an axis passing through its centre and normal to its circular face is*** $mg$ ***equal to the moment of inertia of the same cylinder about an axis passing through its centre and normal to its length, then***




Moment of inertia of a cylinder about an axis passing through centre and normal to circular face $= \frac{MR^2}{2}$

Moment of inertia of a cylinder about an axis passing through centre and normal to its length

$$= M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$$

But $\frac{MR^2}{2} = M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$

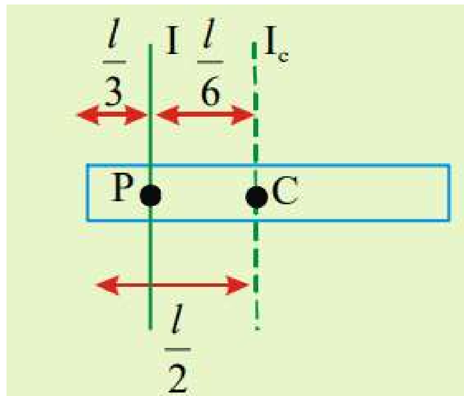
$$\frac{R^2}{2} = \frac{L^2}{12} + \frac{R^2}{4} \Rightarrow \quad \frac{R^2}{4} = \frac{L^2}{12};$$

$$\therefore L = \sqrt{3}R$$

**WE-33:** ***Find the moment of inertia of a thin uniform rod about an axis perpendicular to its length and passing through a point which is at a distance of $\frac{l}{3}$ from one end. Also find radius of gyration about that axis.***

By parallel axes theorem $I = I_C + Mr^2$

$$= \frac{Ml^2}{12} + M\left(\frac{l}{6}\right)^2 = \frac{Ml^2}{9}$$




ii) The radius of gyration, $K = \sqrt{\frac{I}{M}} = \sqrt{\frac{Ml^2}{9M}} = \frac{l}{3}$

**WE-36:** ***The moment of inertia of HCl molecule about an axis passing through its centre of mass and perpendicular to the line joining the $H^+$ and $Cl^-$ ions will be (if the inter atomic distance is $1A^0$).***

$r = 1A^0 = 10^{-10} m$ ; $m_1 = 1amu$ ; $m_2 = 35.5amu$

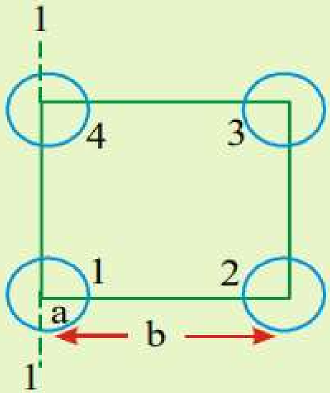
Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2} = 0.9726\, amu$

$\cong 1.624 \times 10^{-27}\, kg \left[\because 1amu = 1.67 \times 10^{-27}\, kg\right]$

Moment of inertia about an axis passing through centre of mass of two particle system and perpendicular to the line joining them is

$$I = \mu r^2 = 1.624 \times 10^{-47}\, kg\, m^2$$

**WE-37:** ***Four solid spheres each of diameter 2a and mass m are placed with their centers on the four corners of a square of side b. Calculate the moment of inertia of the system about any side of the square.***




$$I_1 = \frac{2}{5}ma^2\,;\; I_2 = \frac{2}{5}ma^2 + mb^2$$

$$I_3 = \frac{2}{5}ma^2 + mb^2\,;$$

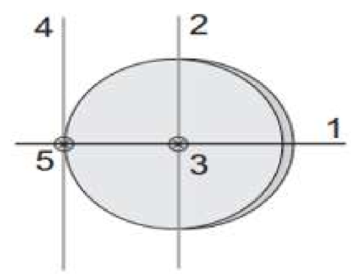
$$I_4 = \frac{2}{5}ma^2$$

Moment of Inertia of the system

$$I = I_1 + I_2 + I_3 + I_4$$

$$= \frac{2}{5}ma^2 + \frac{2}{5}ma^2 + mb^2 + \frac{2}{5}ma^2 + mb^2 + \frac{2}{5}ma^2$$

$$I = \frac{8}{5}ma^2 + 2mb^2$$




$$I_1 = I_2 = \frac{mR^2}{4}$$

$$I_3 = I_1 + I_2 = \frac{mR^2}{2}$$

$$I_4 = I_2 + mR^2 = \frac{5}{4}mR^2$$

$$I_5 = I_3 + mR^2 = \frac{3}{2}mR^2$$

**WE-41:** ***The moment of inertia of a then circular disc about an axis passing through its center and perpendicular to its plane is I. Then, the moment of inertia of the disc about an axis parallel to its diameter and touching the edge of the rim is [E-2008]***

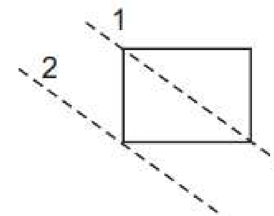
$$I = \frac{MR^2}{2} \Rightarrow MR^2 = 2I$$

$$= \frac{5}{4}MR^2 = \frac{5}{2}I$$

***WE-43:*** ***Two solid spheres A and B each of radius R are made of materials of densities $\rho_A$ and $\rho_B$ respectively. Their moments of inertia about a diameter are $I_A$ and $I_B$ respectively. The value of $I_A/I_B$ is [E-2012]***

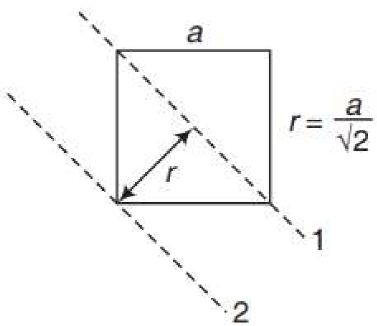
$$\frac{I_A}{I_B}=\frac{\frac{4}{3}\pi R^3\rho_A}{\frac{4}{3}\pi R^3\rho_B}=\frac{\rho_A}{\rho_B}$$

Let $I_1$ and $I_2$ be the moment of inertia of a uniform square plate about axes shown in the figure. Then, the ratio $I_1 : I_2$ is

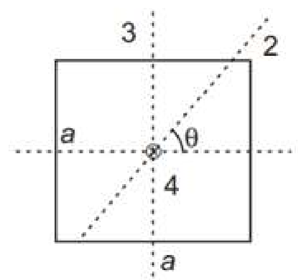



(a) $1:\frac{1}{7}$
(b) $1:\frac{12}{7}$
(c) $1:\frac{7}{12}$
(d) $1:7$

$$I_1=\frac{ma^2}{12}$$




$$I_2=I_1+mr^2$$
$$=\frac{ma^2}{12}+\frac{ma^2}{2}$$
$$=\frac{7}{12}ma^2$$
$$I_1:I_2=1:7$$

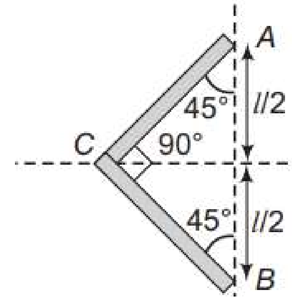


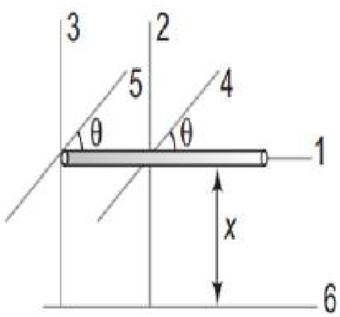

$$I_1=I_2=I_3=\frac{ma^2}{12}$$
$$I_4=I_1+I_3=\frac{ma^2}{6}$$

Linear mass density of the two rods system, $AC$ and $CB$ is $x$. Moment of inertia of two rods about an axis passing through $AB$ is

(a) $\frac{xl^3}{4\sqrt{3}}$ (b) $\frac{xl^3}{\sqrt{2}}$

(c) $\frac{xl^3}{4}$ (d) $\frac{xl^3}{6\sqrt{2}}$



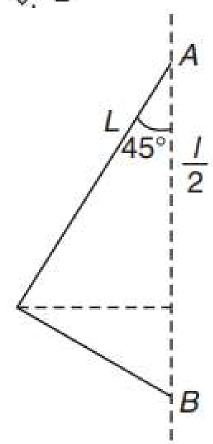

$$I_1 = 0, I_2 = \frac{ml^2}{12}$$

$$I_3 = \frac{ml^2}{3}, I_4 = \frac{ml^2}{12}\sin^2\theta$$

$$I_5 = \frac{ml^2}{3}\sin^2\theta, I_6 = mx^2$$

$$L = \frac{l}{2}\sec 45° = \frac{l}{\sqrt{2}}$$

$$m = Lx = \frac{lx}{\sqrt{2}}$$




$$I = 2\left[\frac{mL^2}{3}\sin^2 45°\right]$$

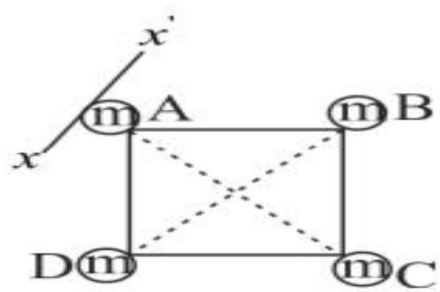
$$= 2\left[\left(\frac{lx}{3\sqrt{2}}\right)\left(\frac{l}{\sqrt{2}}\right)^2\left(\frac{1}{2}\right)\right]$$

$$= \frac{xl^3}{6\sqrt{2}}$$

Four point masses, each of mass m, are placed at the corners of square ABCD of side $l$. The moment of inertia of this system about an axis passing through A and parallel to BD is

(1) $\sqrt{3}ml^2$ (2) $2ml^2$ (3) $ml^2$ (4) $3ml^2$

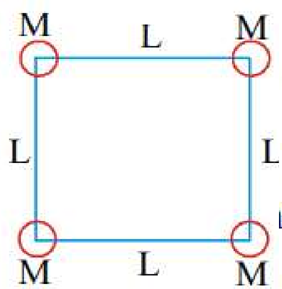
**(4)** The situation is shown in figure.




$$I_{xx'} = 2m \times \left(\frac{l}{\sqrt{2}}\right)^2 + m \times \left(\sqrt{2}l\right)^2 = 3ml^2$$

Four particles each of mass 'm' are placed at the corners of a square of side length '$\ell$'. The radius of gyration of the system about an axis perpendicular to the plane of square and passing through its centre is

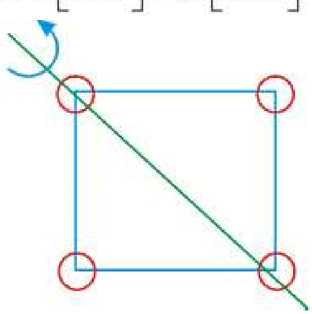
1) $\frac{\ell}{\sqrt{2}}$ 2) $\frac{\ell}{2}$ 3) $\ell$ 4) $\sqrt{2}\ell$




$$4m\left[\frac{l}{\sqrt{2}}\right]^2 = \frac{4ml^2}{2} = 2ml^2$$

$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{2ml^2}{4m}} = \frac{l}{\sqrt{2}}$$

$$I = 2\left[ML^2\right] + M\left[L\sqrt{2}\right]^2; = 2ML^2 + 2ML^2 = 4ML^2$$



$$I = 2\left[M\left(\frac{L}{\sqrt{2}}\right)^2\right] = ML^2$$

In the above problem the moment of inertia of four bodies about an axis perpendicular to the plane of frame and passing through a corner is

1) $ML^2$ 2) $2ML^2$ 3) $2\sqrt{2}ML^2$ 4) $4ML^2$

In above problem the moment of inertia of four bodies about an axis passing through opposite corners of frame is

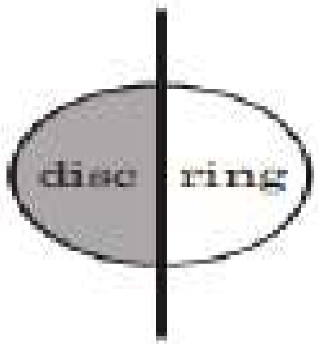
1) $\sqrt{2}ML^2$ 2) $2ML^2$ 3) $ML^2$ 4) $2\sqrt{2}ML^2$

In the above problem the moment of inertia of four bodies about an axis passing through any side of frame is

1) $4ML^2$ 2) $2\sqrt{2}ML^2$ 3) $2ML^2$ 4) $\sqrt{2}ML^2$

$$I = ML^2 + ML^2 = 2ML^2$$

In the shown figure half of the part is disc and other half is a ring both of mass m and radius r. Then moment of inertia of this system about the shown axis is :-

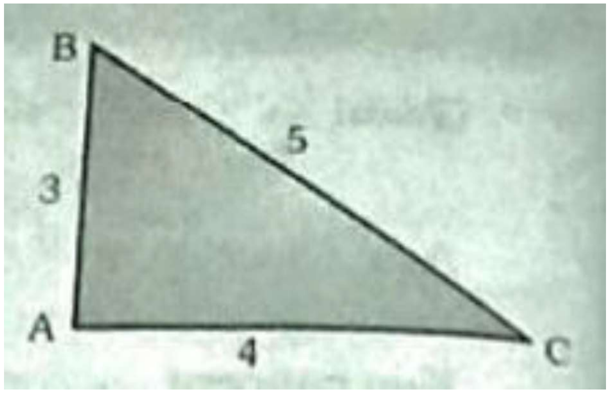



(1) $\frac{3}{4}mr^2$ (2) $\frac{3}{8}mr^2$

(3) $\frac{3}{2}mr^2$ (4) $\frac{mr^2}{4}$

2. Ans. (1)

Sol. $I = \frac{mr^2}{2} + \frac{mr^2}{4} = \frac{3}{4}mr^2$




Triangular lamina का moment of inertia

किस axis के respect में maximum होगा।

ऐसे questions में सबसे छोटी side से pass होने वाली axis के respect में mass distribution (द्रव्यमान वितरण) सबसे ज्यादा दूर तक है जिस कारण $I_{AB}$ maximum होगा। **Note** : $I_{AB} > I_{AC} > I_{BC}$