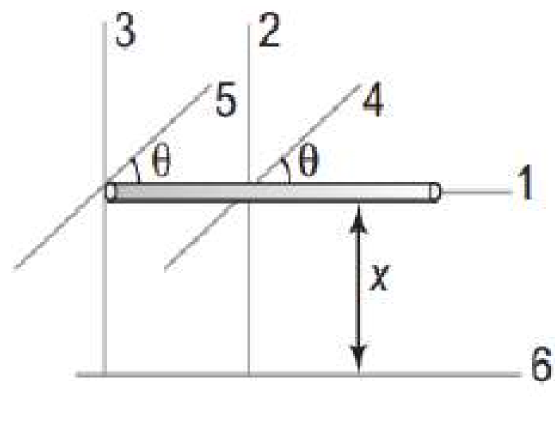
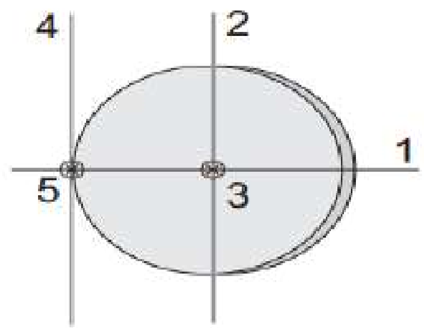
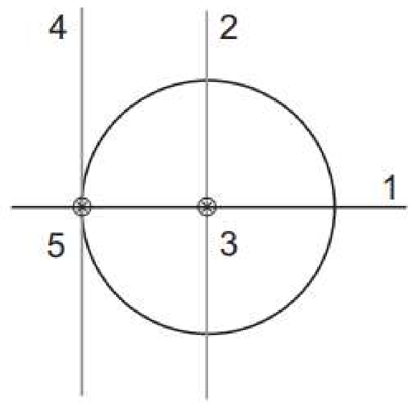
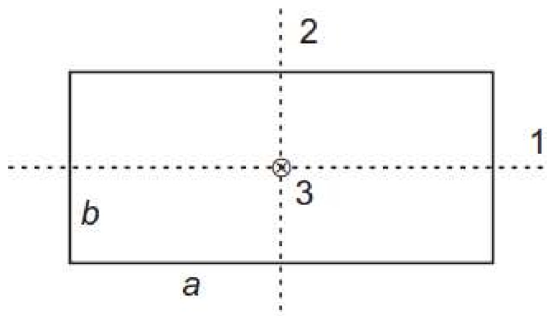
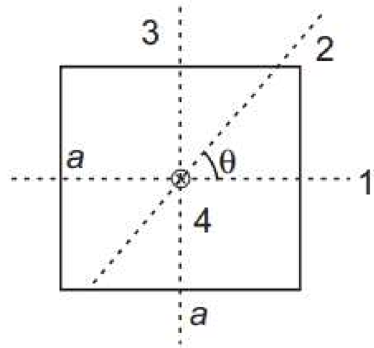
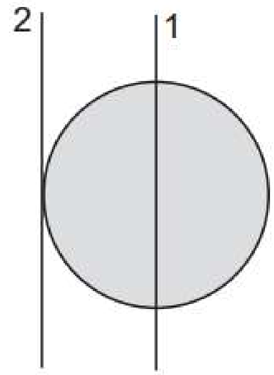
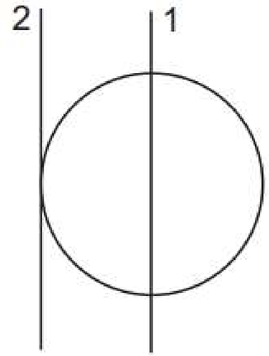
Moment of Inertia of Rigid Bodies

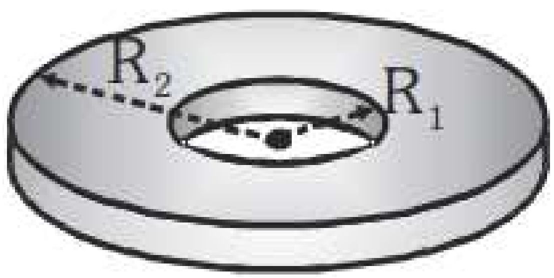
Thin rod		$I_1 = 0, I_2 = \frac{ml^2}{12}$ $I_3 = \frac{ml^2}{3}, I_4 = \frac{ml^2}{12}\sin^2\theta$ $I_5 = \frac{ml^2}{3}\sin^2\theta, I_6 = mx^2$
Circular disc		$I_1 = I_2 = \frac{mR^2}{4}$ $I_3 = I_1 + I_2 = \frac{mR^2}{2}$ $I_4 = I_2 + mR^2 = \frac{5}{4}mR^2$ $I_5 = I_3 + mR^2 = \frac{3}{2}mR^2$

Circular ring		$I_1 = I_2 = \frac{mR^2}{2}$ $I_3 = I_1 + I_2 = mR^2$ $I_4 = I_2 + mR^2 = \frac{3}{2}mR^2$ $I_5 = I_3 + mR^2 = 2mR^2$
Rectangular slab		$I_1 = \frac{mb^2}{12}$ $I_2 = \frac{ma^2}{12}$ $I_3 = I_1 + I_2 = \frac{m}{12}(a^2 + b^2)$

Square slab		$I_1 = I_2 = I_3 = \frac{ma^2}{12}$ $I_4 = I_1 + I_3 = \frac{ma^2}{6}$
Solid sphere		$I_1 = \frac{2}{5}mR^2$ $I_2 = I_1 + mR^2$ $= \frac{7}{5}mR^2$ m = mass of sphere

Hollow sphere		$I_1 = \frac{2}{3}mR^2$ $I_2 = I_1 + mR^2$ $= \frac{5}{3}mR^2$

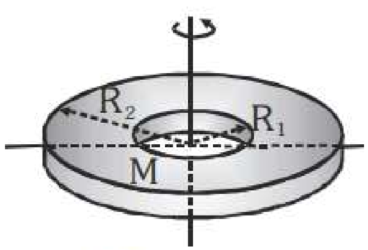
Annular disc



M = Mass

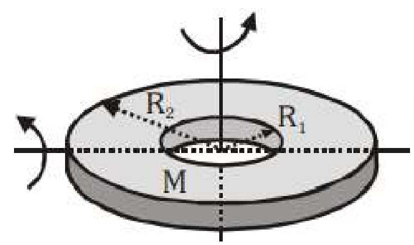
R_1 = Inner Radius

R_2 = Outer Radius



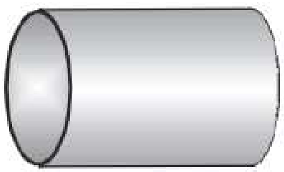
$\frac{M}{2}\left[R_1^2 + R_2^2\right]$

About an axis passing through the centre and perpendicular to the plane of disc



$\frac{M}{4}\left[R_1^2 + R_2^2\right]$

About a diametric axis

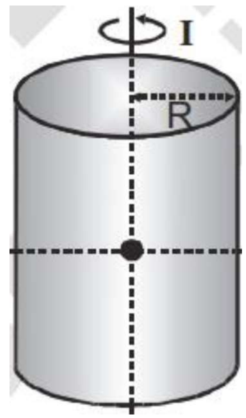


Solid Cylinder

M = Mass

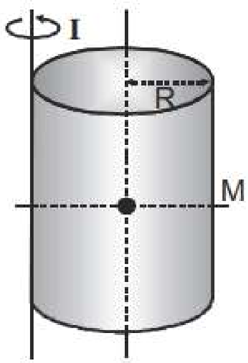
R = Radius

L = Length



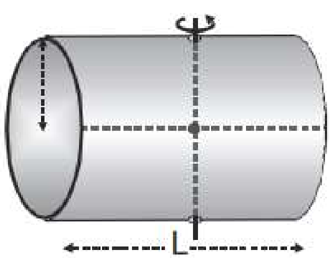
About its geometrical axis which is parallel to its length

$$\frac{MR^2}{2}$$



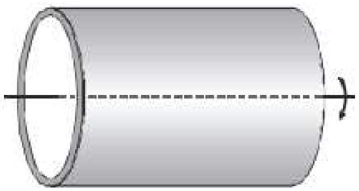
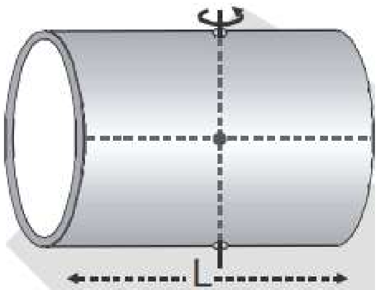
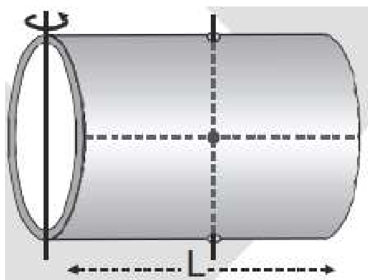
About an axis tangential to the cylindrical surface and parallel to its geometrical axis

$$\frac{3}{2}MR^2$$

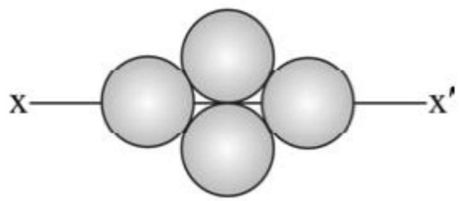


About an axis passing through the centre of mass and perpendicular to its length

$$\frac{ML^2}{12} + \frac{MR^2}{4}$$

…	About its geometrical axis which is parallel to its length	MR^2
	About an axis which is perpendicular to its length and passes through its centre of mass	$\frac{MR^2}{2}+\frac{ML^2}{12}$
	About an axis perpendicular to its length and passing through one end of the cylinder	$\frac{MR^2}{2}+\frac{ML^2}{3}$

The moment of inertia of a sphere (mass M and radius R) about it's diameter is I. Four such spheres are arranged as shown in the figure. The moment of inertia of the system about axis XX' will be :-



(1) 5 I (2) 3 I (3) 9 I (4) 7 I

(3) $I_{XX'} = 2\left(\frac{2}{5}mR^2\right) + 2\left(\frac{2}{5}mR^2 + mR^2\right)$

$= \frac{18}{5}mR^2 = 9\left(\frac{2}{5}mR^2\right)$

Given that $\frac{2}{5}mR^2 = I$

$I_{XX'} = 9I$

Example 12.4 *Consider a uniform rod of mass m and length 2l with two particles of mass m each at its ends. Let AB be a line perpendicular to the length of the rod and passing through its centre. Find the moment of inertia of the system about AB.*

Solution

$$I_{AB} = I_{\text{rod}} + I_{\text{both particles}}$$

$$= \frac{m(2l)^2}{12} + 2(ml^2)$$

$$= \frac{7}{3}ml^2$$

Ans.

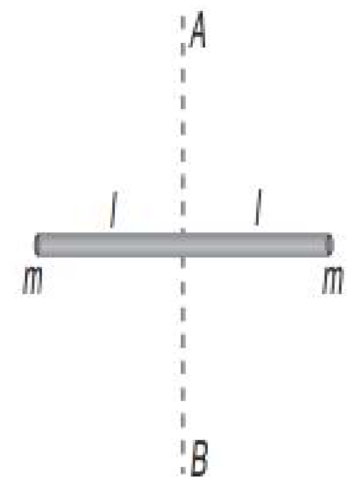


Fig. 12.18

A small part of the rim of a fly wheel breaks off while it is rotating at a constant angular speed. Then its radius of gyration will

(1) Remain unchanged (2) Increase

(3) Decrease

(4) Nothing definite can be said

(3) As mass decreases the moment of inertia also decreases and hence radius of gyration also decreases.

A ring of mass m and radius r is melted and then moulded into a sphere. The moment of inertia of the sphere will be

(1) More than that of the ring

(2) Less than that of the ring

(3) Equal to that of the ring

(4) Exactly one quarter that of the ring

(2) Mass is present at large distance in the ring.

A wheel comprises a ring of radius R and mass M and three spokes of mass m each. The moment of inertia of the wheel about its axis is

(1) $(M+m)R^2$ (2) $\left(M+\frac{m}{4}\right)R^2$

(3) $\left(\frac{M+m}{2}\right)R^2$ (4) $(M+3m)R^2$

(1) $I = I_{ring} + I_{spokes}$

$= MR^2 + 3\left(\frac{mR^2}{3}\right)$

$= (M+m)R^2$