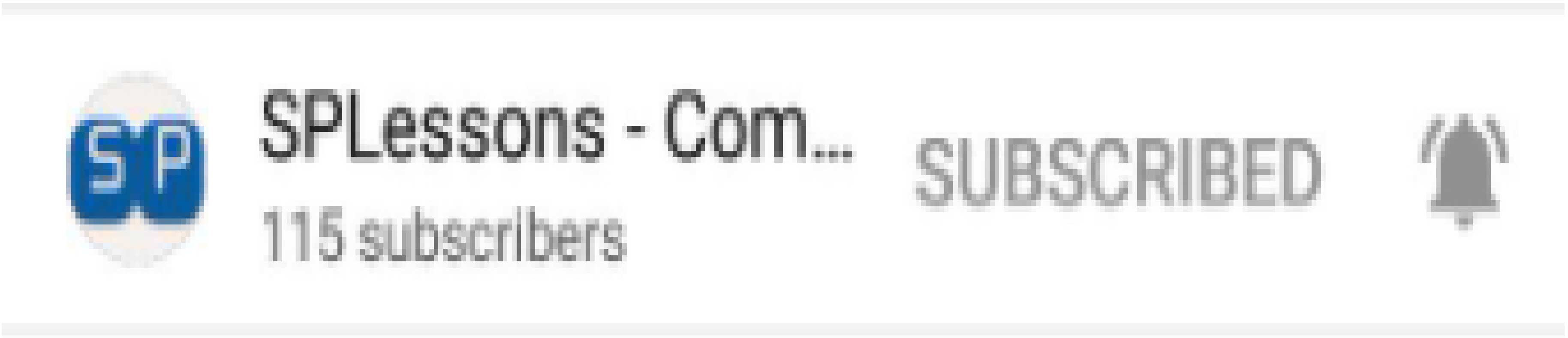

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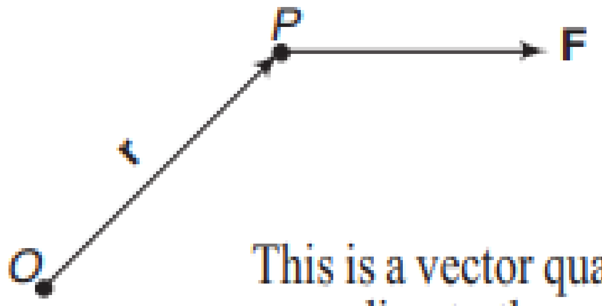
ALS0 USEFUL FOR BOARD EXAMS AND NEET

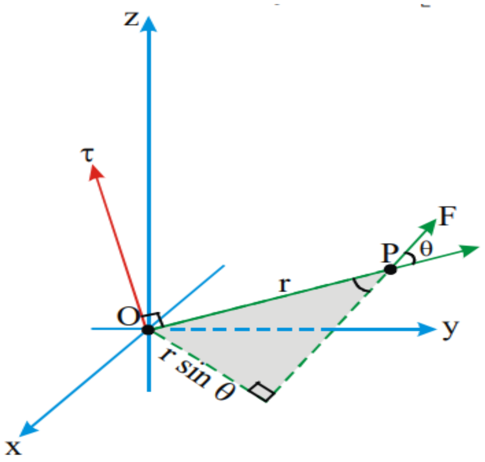
Moment of force (Torque)

Suppose a force **F** is acting on a particle *P* and let **r** be the position vector of this particle about some reference point *O*. The torque of this force **F**, about *O* is defined as,



$$\tau = F(r\sin\theta) \Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$$

S.I. Unit: Nm ***Dimensional formula:*** $[ML^2T^{-2}]$.



This is a vector quantity having its direction perpendicular to both **r** and **F**, according to the rule of cross product.

Note *Here,* $\mathbf{r} = \mathbf{r}_P - \mathbf{r}_O$

$\mathbf{r}_P$ = *position vector of point, where force is acting and*

$\mathbf{r}_O$ = *position vector of point about which torque is required.*

Application: A force of given magnitude applied at right angles to the door at its outer edge is most effective in producing rotation.

- ➢ The moment of a force vanishes if either the magnitude of the force is zero, or if the line of action of the force passes through the fixed point.
- ➢ If the direction of **F** is reversed, the direction of the moment of force is also reversed.

- If directions of both **r** and **F** are reversed, the direction of the moment of force remains the same.

Sign convention : Torque that produces anti clockwise rotation is taken as positive and clockwise rotation taken as negative.

Example 12.7 *Find the torque of a force* $\mathbf{F} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$ *N about a point O. The position vector of point of application of force about O is* $\mathbf{r} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ *m*.

Solution Torque $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 1 & 2 & -3 \end{vmatrix}$

$$= \hat{\mathbf{i}}(-9+2) + \hat{\mathbf{j}}(-1+6) + \hat{\mathbf{k}}(4-3)$$

or $\tau = (-7\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}})$ N-m **Ans.**

Example 12.8 *A small ball of mass 1.0 kg is attached to one end of a 1.0 m long massless string and the other end of the string is hung from a point O. When the resulting pendulum is making 30° from the vertical, what is the magnitude of net torque about the point of suspension?*
[Take $g = 10\ m/s^2$*]*

Solution Two forces are acting on the ball

(*i*) tension (T) (*ii*) weight (mg)

Torque of tension about point O is zero, as it passes through O.

$$\tau_{mg} = F \times r_{\perp}$$

Here, $r_{\perp} = OP = 1.0 \sin 30° = 0.5 \text{ m}$

$\therefore$ $\tau_{mg} = (mg)(0.5)$

$= (1)(10)(0.5) = 5 \text{ N-m}$ **Ans.**

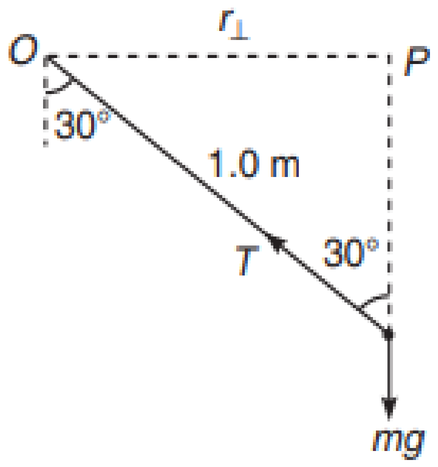


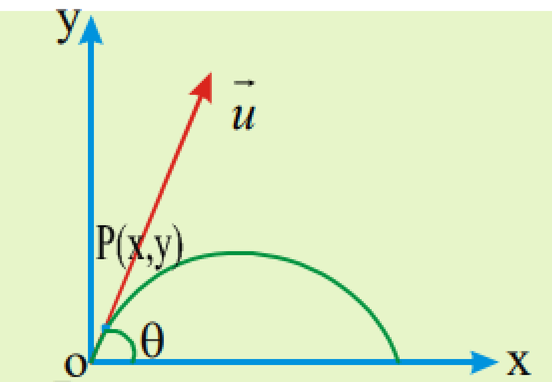
Fig. 12.38

Example 12.9 *A force* $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})$ *N is acting at point P(2 m, –3 m, 6 m). Find torque of this force about a point O whose position vector is* $(2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ *m.*

Solution $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ Here, $\mathbf{r} = \mathbf{r}_P - \mathbf{r}_O = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = (2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ m

Now, $$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & 3 \\ 2 & 3 & -4 \end{vmatrix} = (-17\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}})\ \text{N-m}$$ **Ans.**

WE-22: ***A particle is projected at time t=0 from a point 'O' with a speed 'u' at an angle 'θ' to horizontal. Find the torque of a gravitational force on projectile about the origin at time 't'.(x, y plane is vertical plane)***



Sol. $$\vec{r} = (u\cos\theta)t\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

$$\vec{F} = -(mg)\hat{j} \quad ; \quad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \left[(u\cos\theta)t\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}\right] \times mg\left(-\hat{j}\right)$$

$$\vec{\tau} = (u\cos\theta)t(mg)\left(\hat{i} \times -\hat{j}\right)$$

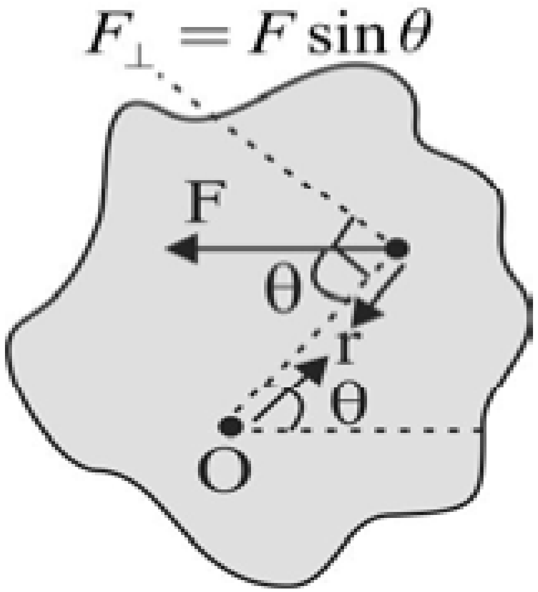
$$\tau = -mg\,(u\cos\theta)t\left(\hat{k}\right)$$

The magnitude of torque can be calculated as

$$\tau = rF_{\perp}$$

r- distance drawn from the point of measurement to the point where the force F acts.

$F_{\perp}$ - is the component of force present perpendicular to the radial line.



Here the torque is measured about point O.

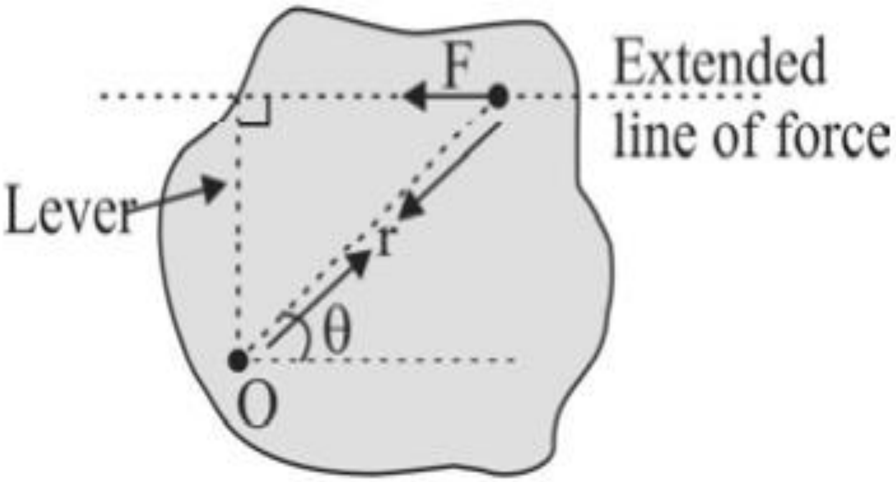
$$\tau_0 = Fr\sin\theta$$

- The magnitude of torque can also be determined as

$$\tau_O = (lever)F$$

F - is the magnitude of the force.

Lever - is the perpendicular distance drawn from point O to the extended line of force.



$$\text{Lever} = r\sin\theta$$

$$\tau_O = Fr\sin\theta$$

- Torque is a pseudo vector.

Turning effect is produced by

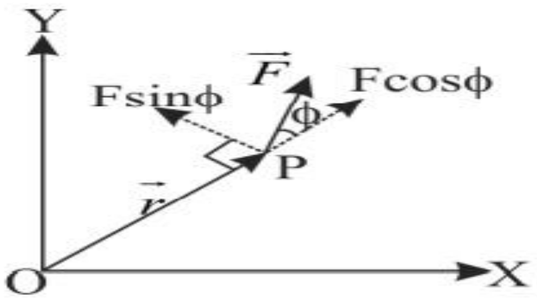
(1) Radial component of force

(2) Transverse component of force

(3) Both radial & transverse components of force

(4) None of the above

(2) Consider a force F that acts at point P. The torque produced by the force about O is



$\tau = r\ F \sin\phi$

$F \sin\phi$ is the transverse component of force correct option is (2)

Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ *and* $\vec{T}$ be the torque of this force about the origin.

(1) $\vec{r}.\vec{T} = 0\ and\ \vec{F}.\vec{T} = 0$

(2) $\vec{r}.\vec{T} \neq 0\ and\ \vec{F}.\vec{T} = 0$

(3) $\vec{r}.\vec{T} = 0\ and\ \vec{F}.\vec{T} \neq 0$

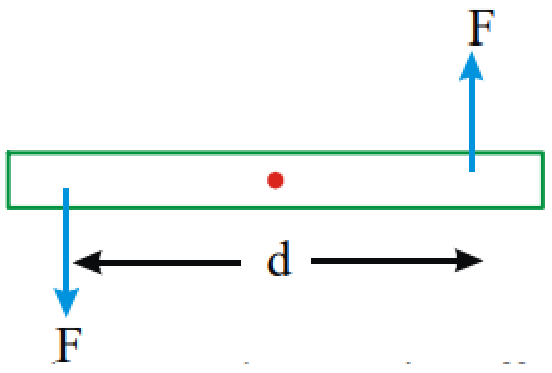
(4) $\vec{r}.\vec{T} \neq 0\ and\ \vec{F}.\vec{T} \neq 0$

(1) $\vec{T} = \vec{r} \times \vec{F}$

$\vec{T}$ *is* $\perp$ *to both* $\vec{r}$ *and* $\vec{F}$ *so* $\vec{r}.\vec{T} = 0\ and\ \vec{F}.\vec{T} = 0$

Moment of couple:

A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation. If an object is not on pivot (unconstrained) a couple causes the object to rotate about its centre of mass.



This couple can produce turning effect (or) torque on the body. Moment of couple is a measure of turning effect (τ).

$\therefore \tau$ = moment of couple=magnitude of either force $\times$ perpendicular distance between the forces

$\therefore \tau = Fd$

Two equal and opposite forces act on a rigid body at a certain distance. Then

(1) The body may rotate about its centre of mass.

(2) The body is in equilibrium.

(3) The body cannot rotate about its centre of mass.

(4) The body may rotate about any point other than its centre of mass.

(1) Net force on centre of mass is zero, i.e., the centre mass cannot move at all. Hence, the body may rotate about centre of mass.

A door 1.6 m wide requires a force of 1 N to be applied at the free end to open or close it. The force that is required at a point 0.4 m distant from the hinges for opening or closing the door is

(1) 2.4 N (2) 4 N (3) 1.2 N (4) 3.6 N

A particle of mass $m = 1$kg is projected with speed $u = 20\sqrt{2}$ m/s at angle $\theta = 45°$ with horizontal. Find the torque of the weight of the particle about the point of projection when the particle is at the highest point.