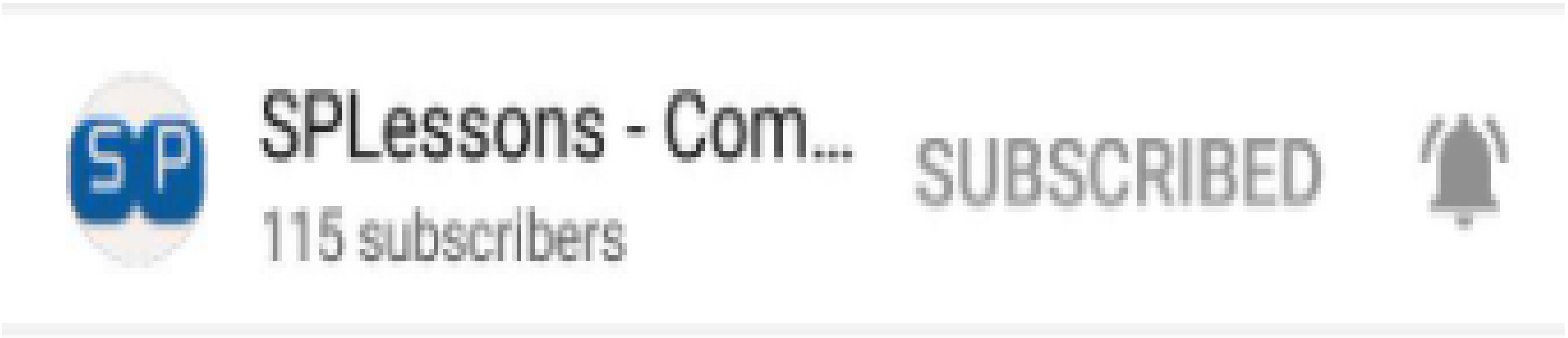

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ALS0 USEFUL FOR BOARD EXAMS AND NEET

Moment of inertia [Rotational Inertia]:

- A body at rest cannot start rotating itself or a rotating body cannot stop rotating on its own. Hence, a body has inertia of rotational motion.
- The quantity measuring the inertia of rotational motion is known as moment of inertia.
- Moment of inertia of a particle of mass m is

$$I = mr^2$$

Where r = perpendicular distance of particle from axis of rotation.

S.I unit: kgm^2 ; Its D.F - ML^2

- Moment of inertia of a group or system of particles is $I = m_1r_1^2 + m_2r_2^2 + ...m_nr_n^2$ $I = \Sigma mr^2$

Where $m_1, m_2.........m_n$ are masses of particles and $r_1, r_2..........r_n$ are their perpendicular distances from axis of rotation.

- Moment of Inertia in rotational motion is analogous(similar) to mass in translatory motion.
- Moment of Inertia of a rigid body depends on the following three factors.

a) mass of the body b) position of axis of rotation

c) Nature of distribution of mass.

Note-1: Moment of inertia of a rotating rigid body is independent of its angular velocity.

Note-2: Moment of inertia of a metallic body depends on its temperature.

Radius of Gyration(K): Radius of gyration of a rigid body about an axis of rotation is distance between the axis of rotation and a point at which the whole mass of the body can be supposed to be concentrated so that its moment of inertia would be the same with the actual distribution of mass.

- Moment of inertia of a rigid body of mass M is

$I = MK^2$

Where K = radius of gyration

$$K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^n}{n}}$$

Where n is total number of particles in the body and $r_1, r_2............r_n$ are their perpendicular distances from axis of rotation.

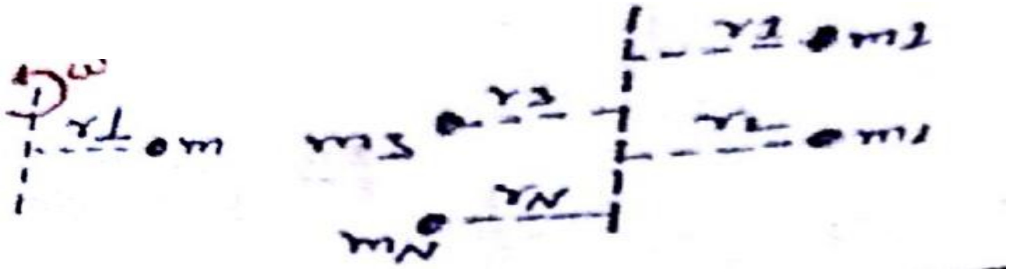
S.I unit: metre ***CGS unit:*** cm

Dimensional formula: $[M^0LT^0]$

Note: K is not the distance of centre of mass of body from the axis considered.

- **Radius of gyration of a rigid body depends on the following two factors**
  a) Position of axis of rotation.
  b) Nature of distribution of mass.

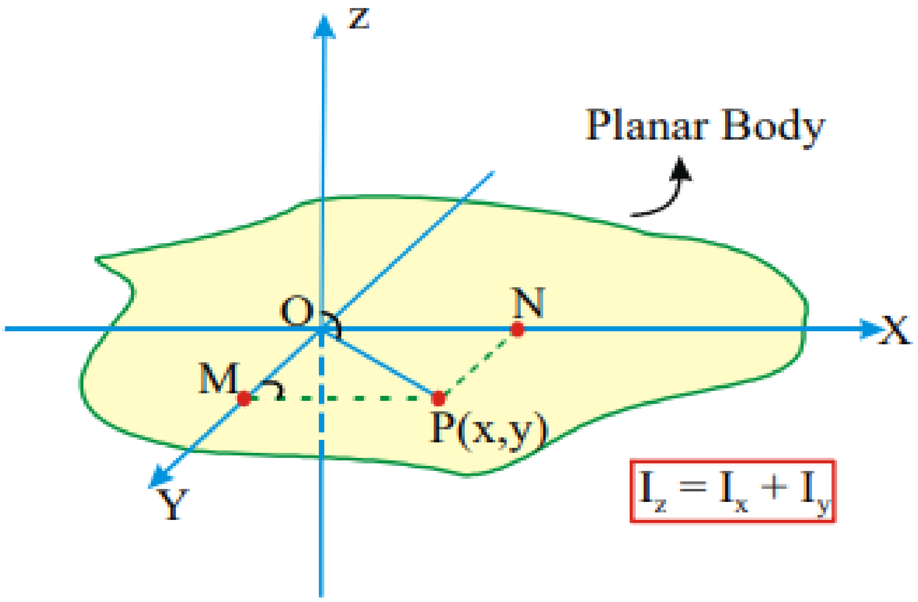
M.O.I Not depend on -
* Angular disp.
* Angular velocity.
* Angular Accln.
* Torque
* || dimenson of Axis of Rotation.

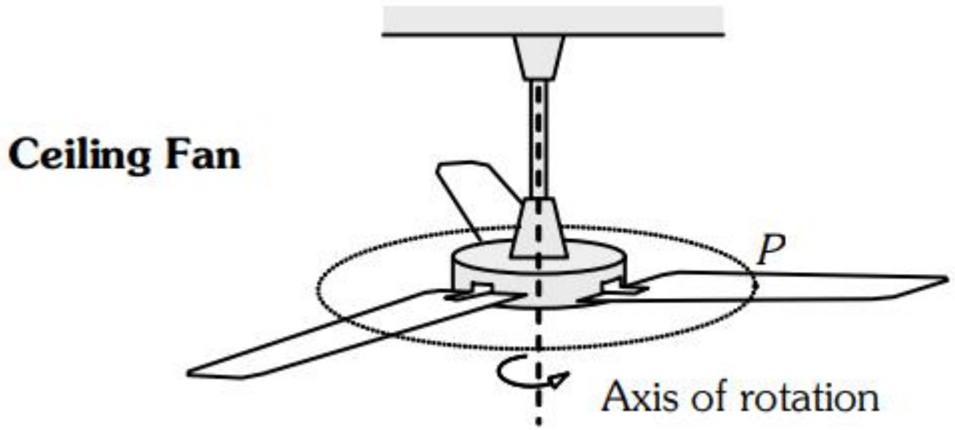


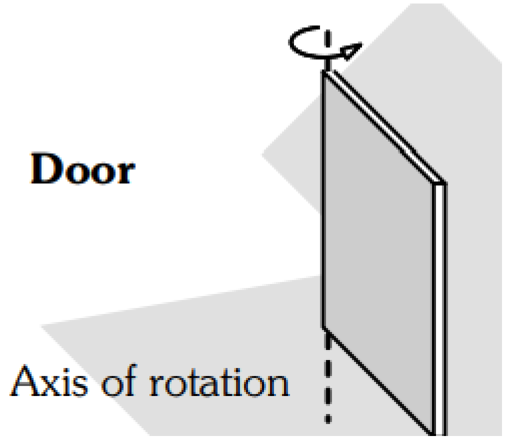
Perpendicular axes theorem

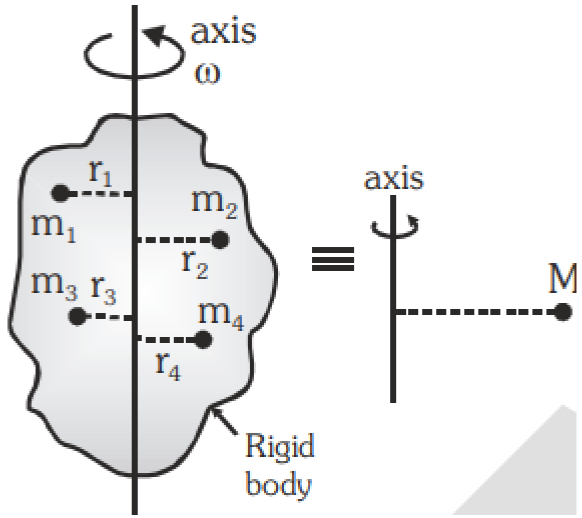
Statement: It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two mutually perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

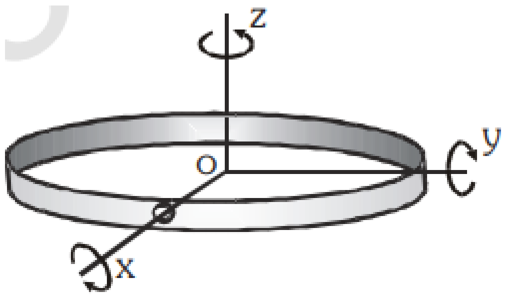
- This theorem is applicable to bodies which are planar.
- This theorem applies to flat bodies whose thickness is very small compared to their other dimensions.
- $K_z = \sqrt{K_x^2 + K_y^2}$

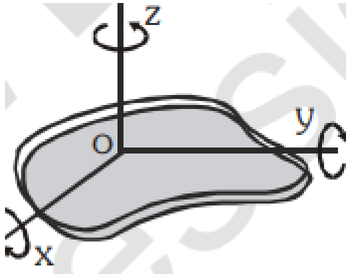


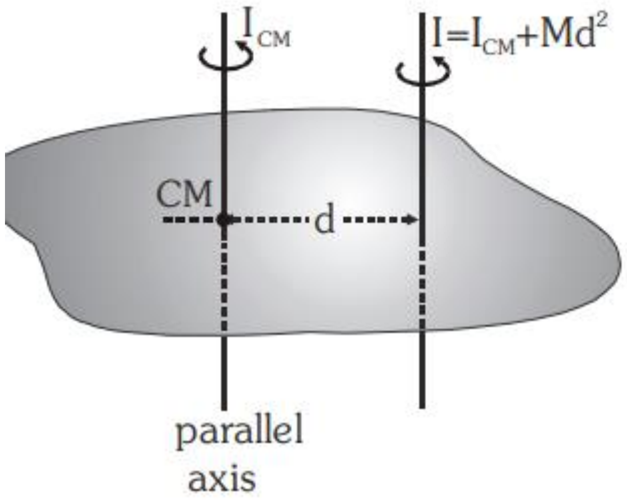

Ceiling Fan
P
Axis of rotation


Door
Axis of rotation


axis
ω
r_1
m_1
m_2
r_2
m_3 r_3
m_4
r_4
Rigid body
≡
axis
M

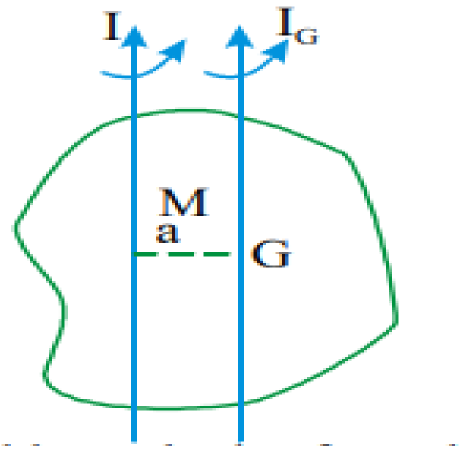

z
y
O
x


z
y
O
x



Statement: The moment of inertia of a body about an axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of gravity and the product of its mass and the square of the distance between the two parallel axes.

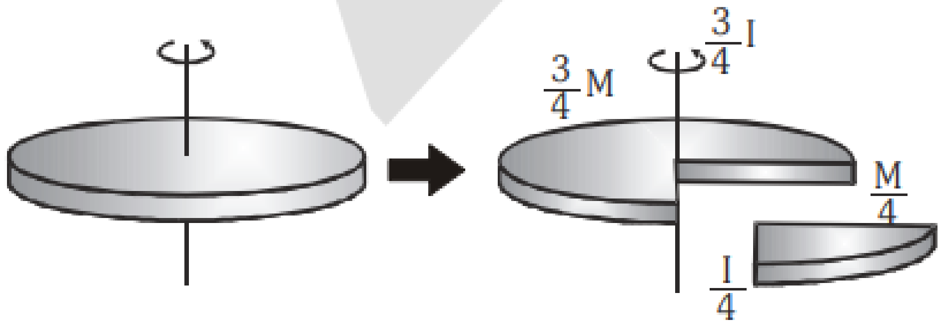
- This theorem is applicable to a body of any shape.



$$\boxed{I = I_G + Ma^2}$$

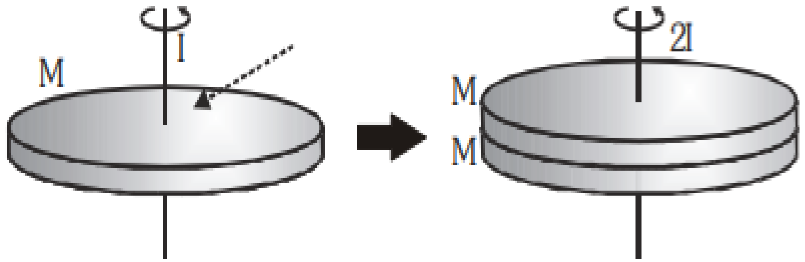
- $K = \sqrt{K_G^2 + a^2}$

For Symmetrical separation radius of gyration remains unchanged.



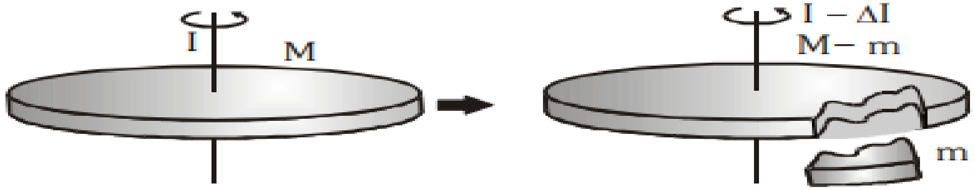
$\Rightarrow$ K remains unchanged

- For Symmetrical attachment radius of gyration remains unchanged.



$\Rightarrow$ K remains unchanged

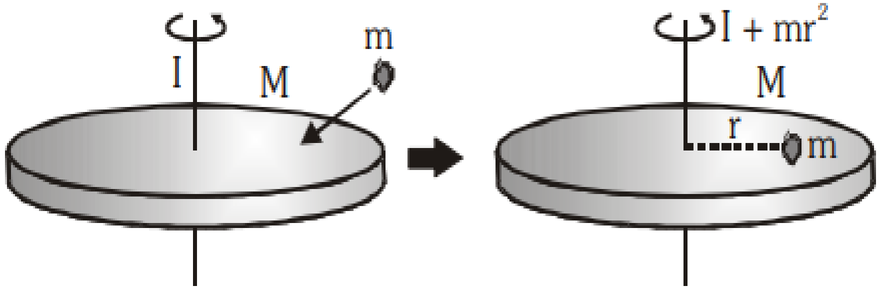
- For asymmetrical separation radius of gyration will change.



$\Rightarrow \quad K = \sqrt{\frac{I - \Delta I}{M - m}} \quad \Rightarrow$ K change

Here ΔI = M. I. of detached mass w.r.t. same axis.

For asymmetrical attachment radius of gyration will be changed.



$$\Rightarrow \quad K = \sqrt{\frac{I + mr^2}{M + m}}$$

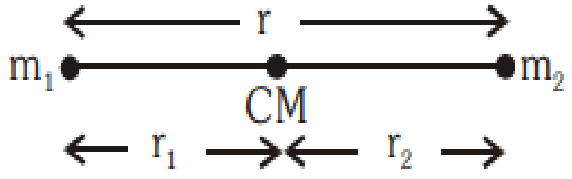
Moment of inertia is not a vector as direction CW or ACW is not to be specified and also not a scalar as it has different values in different directions (i.e. about different axes). It is a tensor quantity.

If a wheel is to be made by using two different materials then for larger moment of inertia, larger density material should lie in the periphery .

* If obj. ⊕nt in a 'x-y' plane ⇒ $I_x + I_y \Rightarrow I_z$
* If obj. ⊕nt in a 'y-z' plane ⇒ $I_y + I_z \Rightarrow I_x$
* ... ⊕nt in a 'z-x' plane ⇒ $I_y = I_z + I_x$

Two masses m_1 and m_2 are placed at a separation r. Find out the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the line joining the masses.

Solution.



$$m_1 r_1 = m_2 r_2 \text{ and } r_1 + r_2 = r \Rightarrow r_1 = \frac{m_2 r}{m_1 + m_2},\ r_2 = \frac{m_1 r}{m_1 + m_2}$$

Moment of inertia $I = m_1 r_1^2 + m_2 r_2^2 = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2$

Note : Here $I = \mu r^2$ where μ (reduced mass) $= \dfrac{m_1 m_2}{m_1 + m_2}$.

** Standard

* For two mass system

$\mu = \frac{m_1 m_2}{m_1 + m_2} \Rightarrow \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

* For Three mass system.

$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}$

* For 'N' Mass system

$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} + \cdots + \frac{1}{m_N}$

Analogy between Linear Motion & Rotational Motion			
Linear Motion		**Rotational Motion**	
Quantities			
Displacement	s	Angular displacement	θ
Velocity	v	Angular velocity	ω
Acceleration	a	Angular acceleration	α
Force	F	Torque	τ
Mass	m	Moment of inertia	I
Expressions			
Velocity	$v = \frac{ds}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Force	$F = ma = \frac{d}{dt}(mv)$	Torque	$\tau = I\alpha = \frac{d}{dt}(I\omega)$
Work done	$W = Fs$	Work done	$W = \tau\theta$
Linear K.E.	$E = \frac{1}{2}mv^2$	Rotational K.E.	$E = \frac{1}{2}I\omega^2$
Power	$P = Fv$	Power	$P = \tau\omega$
Linear momentum	$p = mv$	Angular momentum	$P = I\omega$
Impulse	$Ft = mv - mu$	Angular Impulse	$\tau\Delta t = I\omega_f - I\omega_i$

Equations of motion

$v - u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 - u^2 = 2as$	$\omega^2 - \omega_0^2 = 2\alpha\theta$
$s_n = u + \frac{1}{2}a(2n-1)$	$\theta_n = \omega_0 + \frac{1}{2}\alpha(2n-1)$